Transportation networks and Voronoi Diagrams

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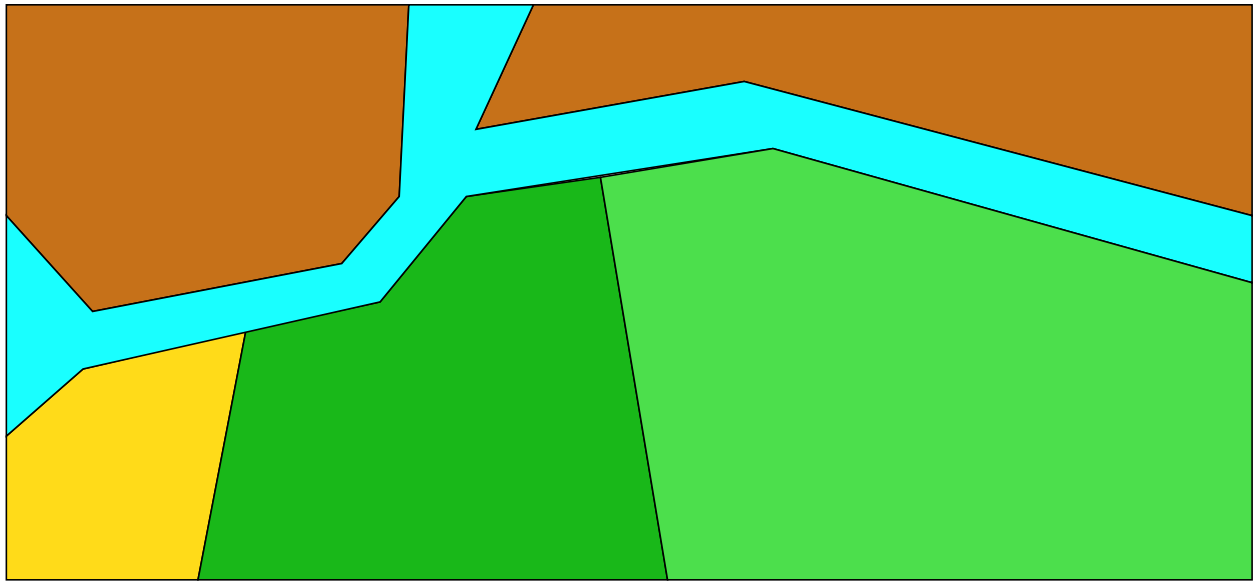
Time metrics

- Different speeds on the plane



Time metrics

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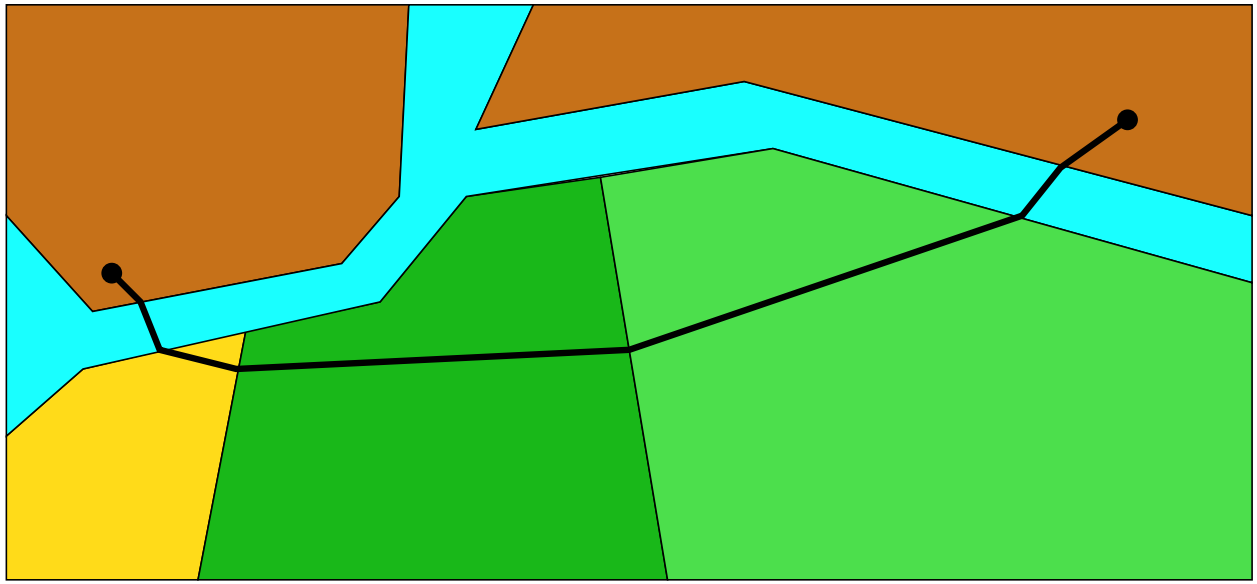
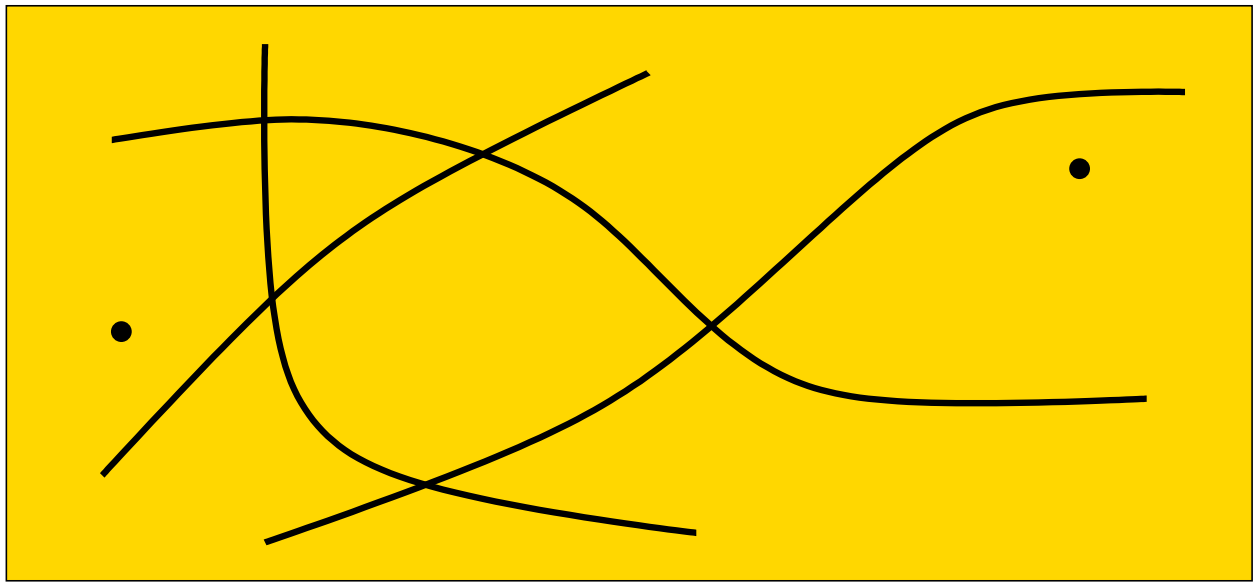


Time metrics

- Different speeds on the plane
- Transportation networks



Transportation Networks

- How do time distances behave?
- How can we compute shortest paths?
- Can we efficiently compute bisectors?
- What kind of Voronoi Diagrams do we obtain?

Transportation Models

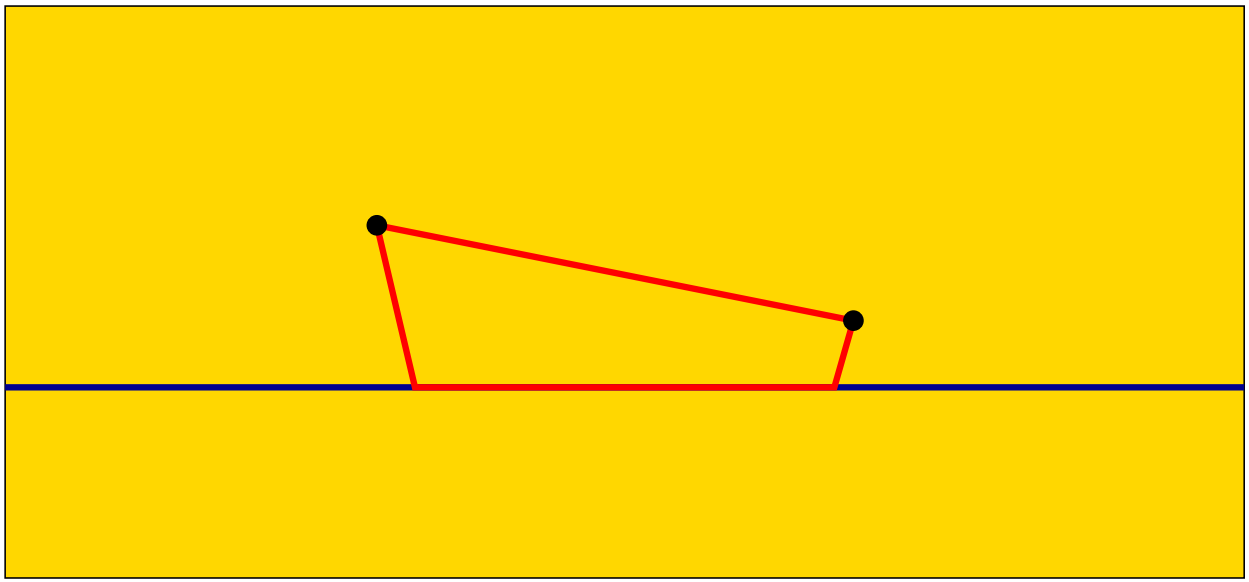
- Continuous line network
- Continuous wedge network
- Continuous circle network
- Continuous model with isothetical restrictions
- Discrete Model

Line Model: Description

- Transportation network is a single straight line L
- Speed on the plane is 1 and speed on L is $v >> 1$
- Travellers can move in the plane in any direction
- Every point in L is an access point to it

Line Model: Description

- **Shortest paths**
- Time circles
- Walking region
- Chosen bisectors



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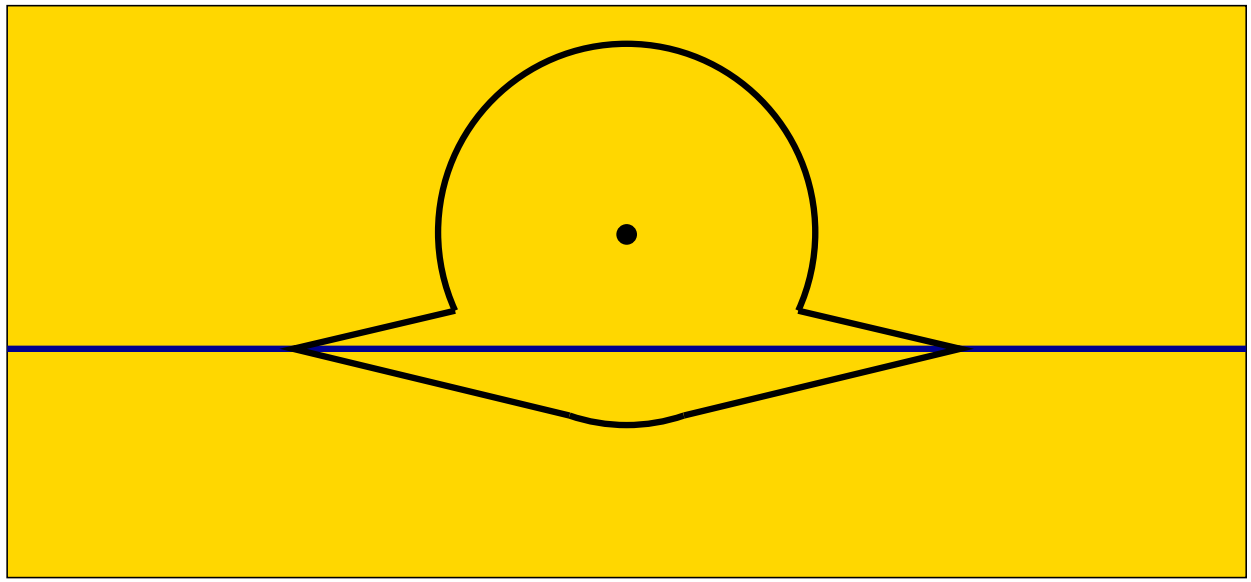

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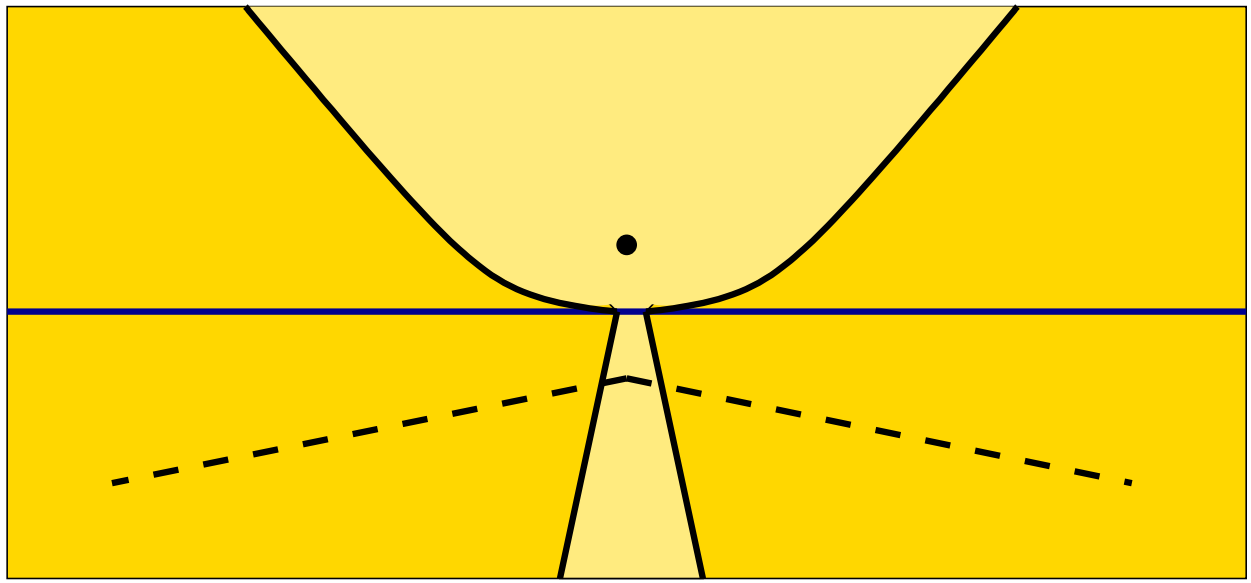

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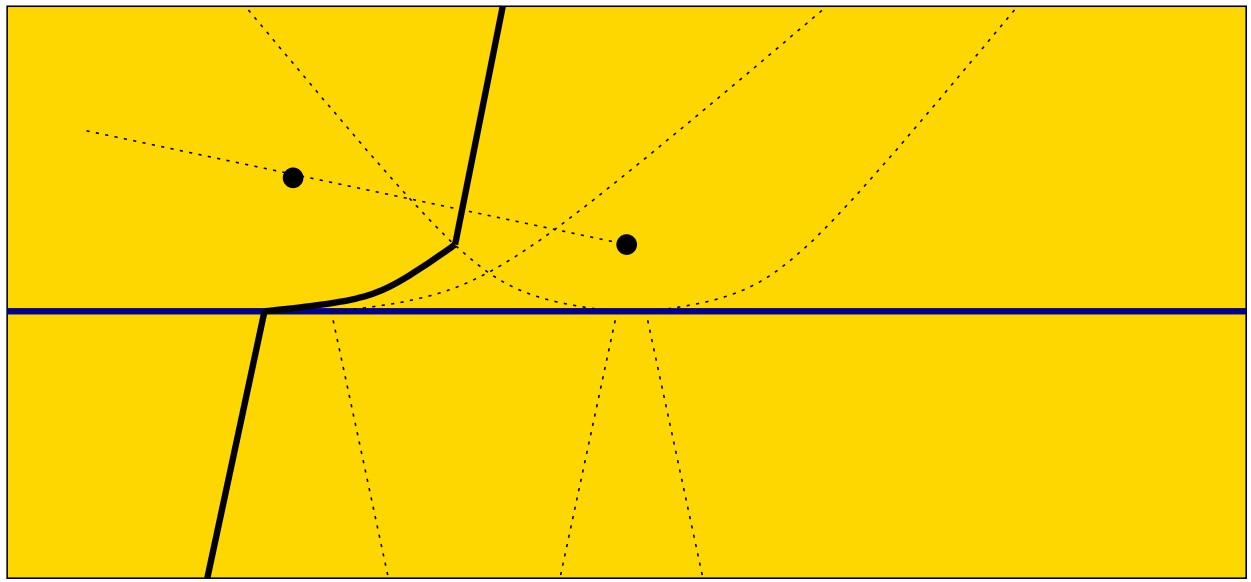

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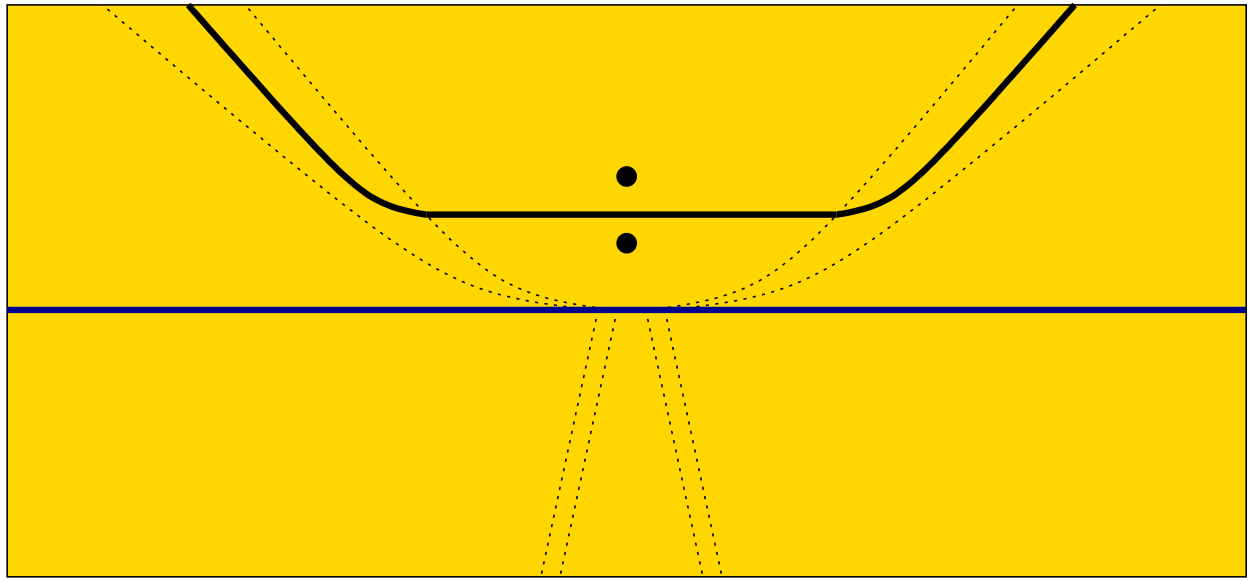
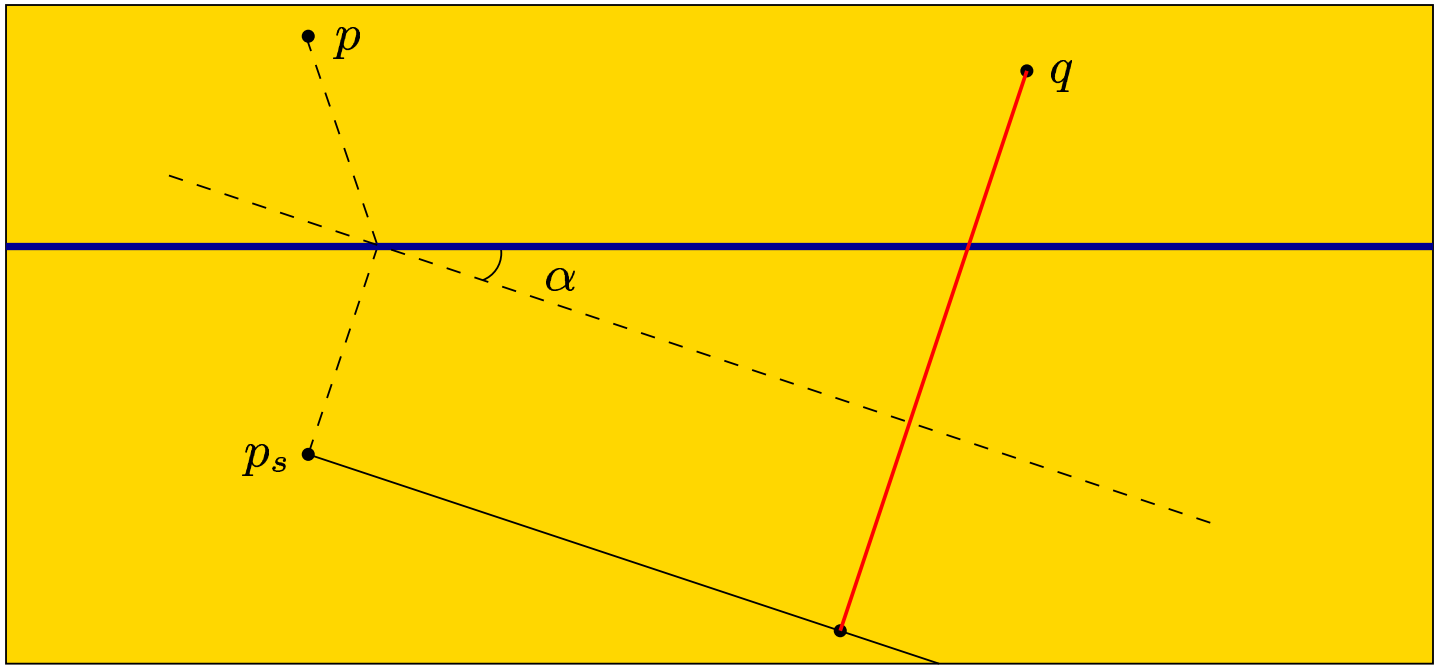


Line Model: Time Voronoi diagram

- The Time Voronoi diagram can be constructed as points and lines Voronoi diagram in $O(n \log n)$ time using linear space



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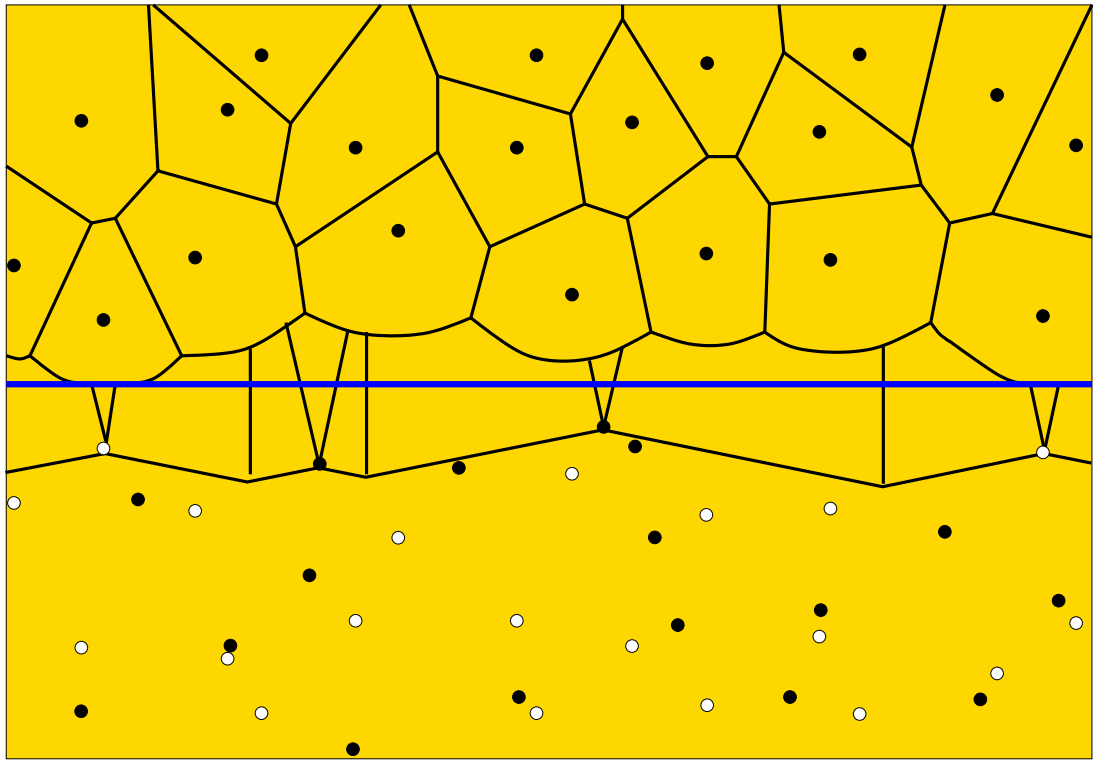
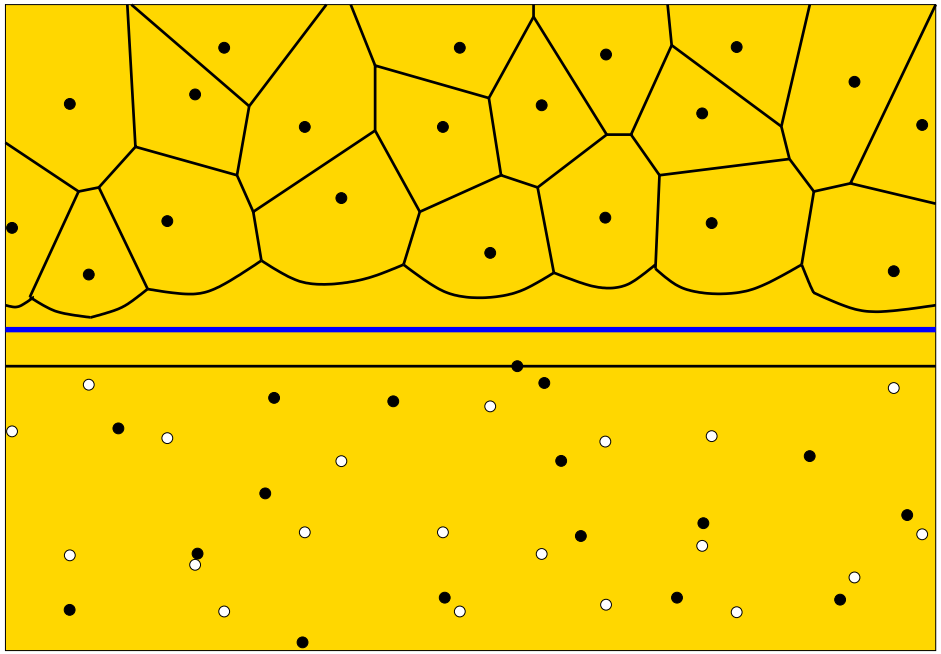


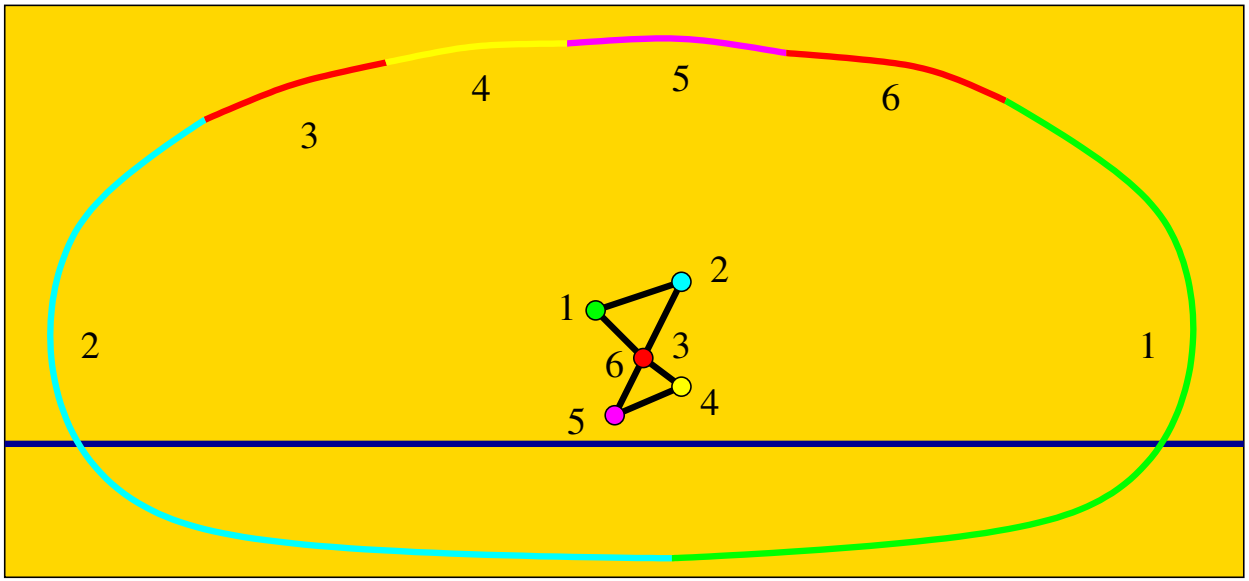
High speed Line Model: Time Voronoi diagram

- **Theorem:** Upper and lower time Voronoi diagrams are unfinished executions of Fortune's algorithm
- **Corollary:** The time Voronoi Diagram for a set of n points under the time distance when the speed tends to infinity can be computed in $O(n \log n)$ time and linear space



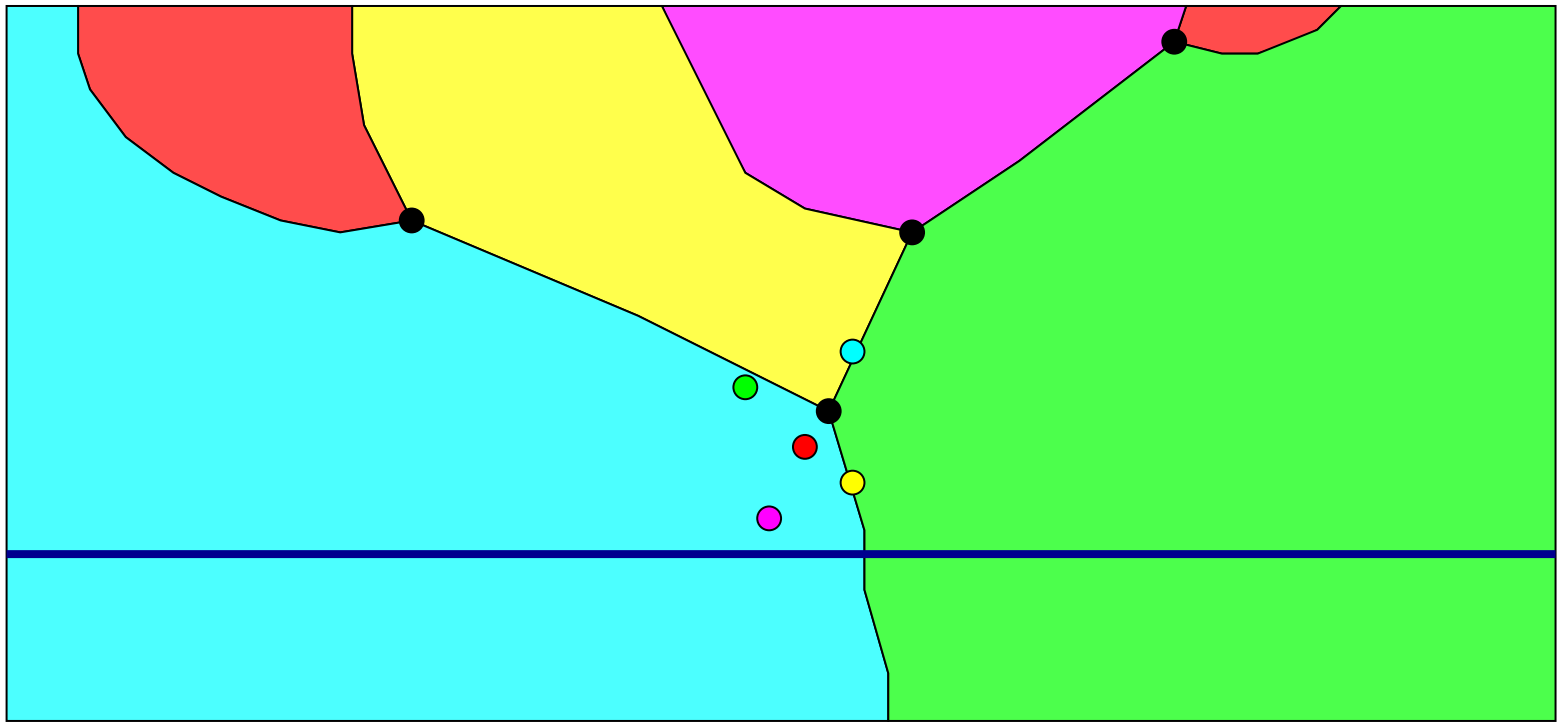
Line Model: Farthest time Voronoi diagram

- Points with non empty Voronoi region
  - belong to the Euclidean convex hull
  - are not dominated from the left or from the right
- All non empty regions intersect some curve enclosing the set far enough from it in a certain order



Line Model: Farthest time Voronoi diagram

- Nodes of the farthest time Voronoi diagram can be ordered depending on their time distance to the points



Line Model: Farthest time Voronoi diagram

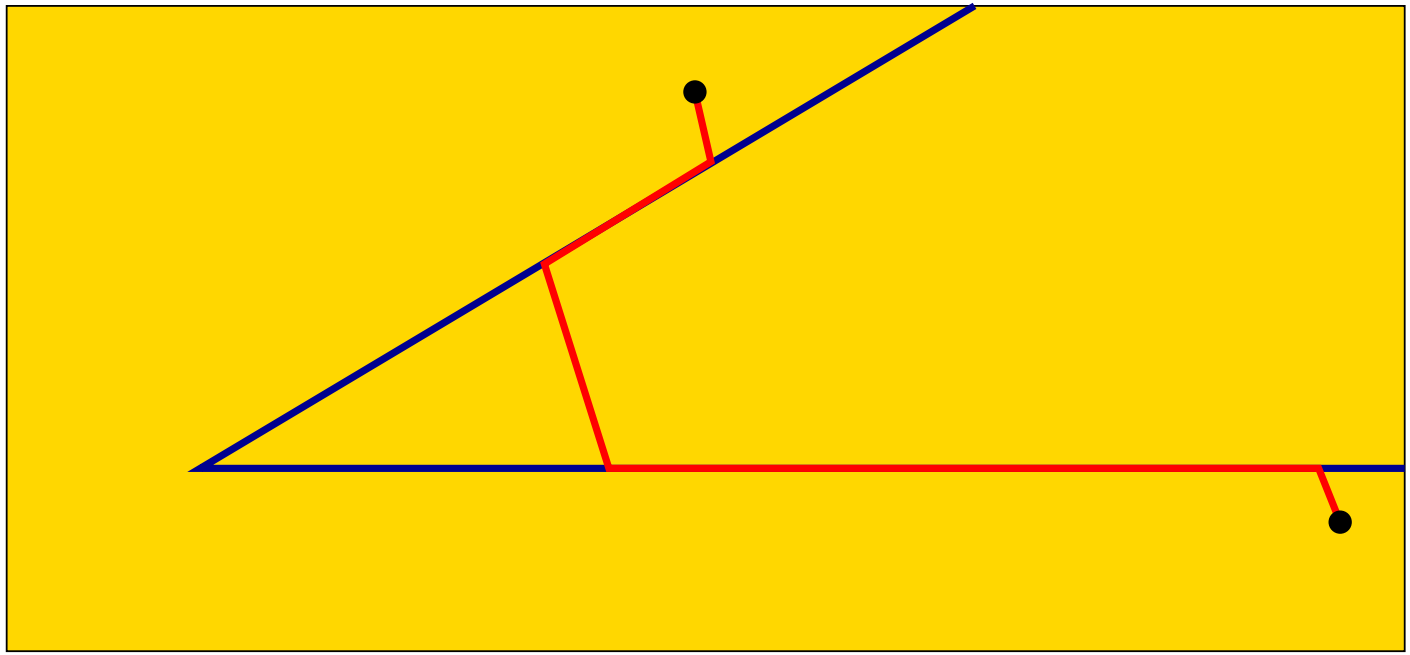
- **Theorem:** The ordered sequence of non empty regions can be computed in $O(n \log n)$ time and linear space
- **Theorem:** The farthest time Voronoi diagram for a set of n points can be computed in $O(n \log n)$ time and linear space with an ”ear cutting” process, given the ordered sequence of nonempty regions

Wedge Model: Description

- Transportation network is a wedge W with interior angle β
- Speed on the plane is 1 and speed on W is $v >> 1$
- Travellers can move in the plane in any direction
- Every point in W is an access point to it

Wedge Model: Description

- **Shortest paths**
- Time circles
- Chosen bisectors



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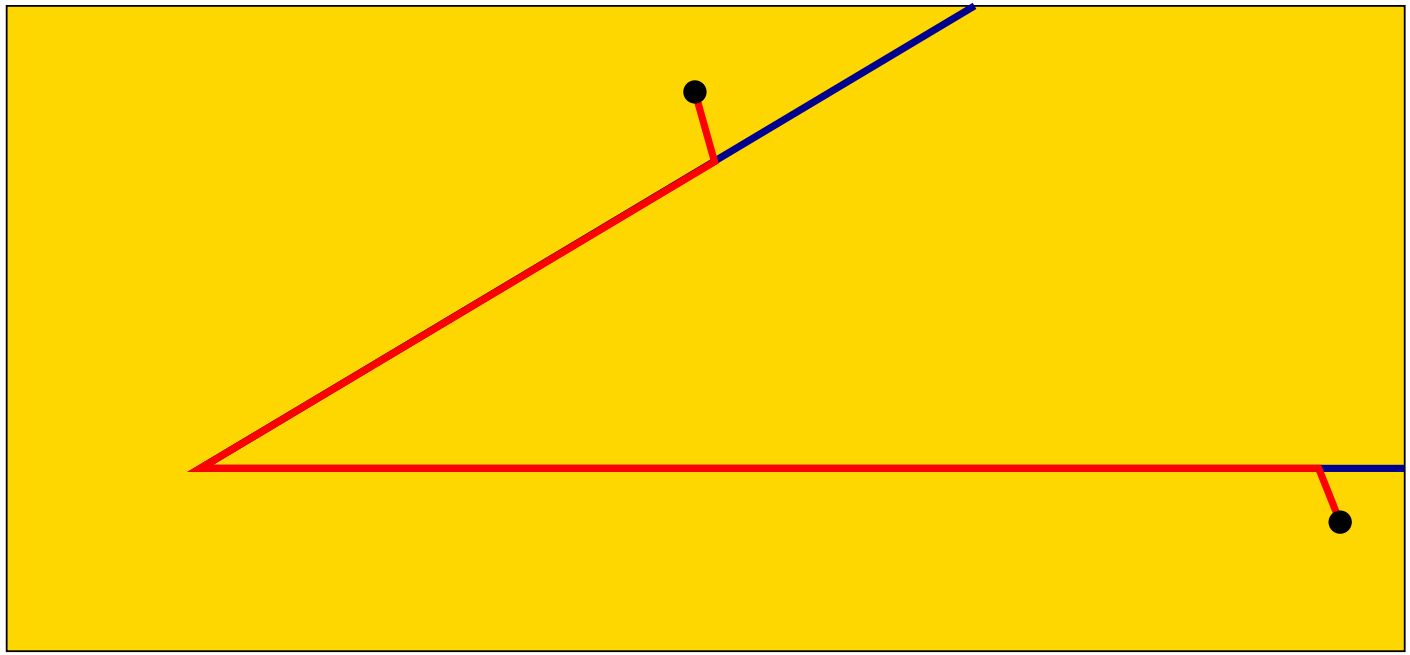

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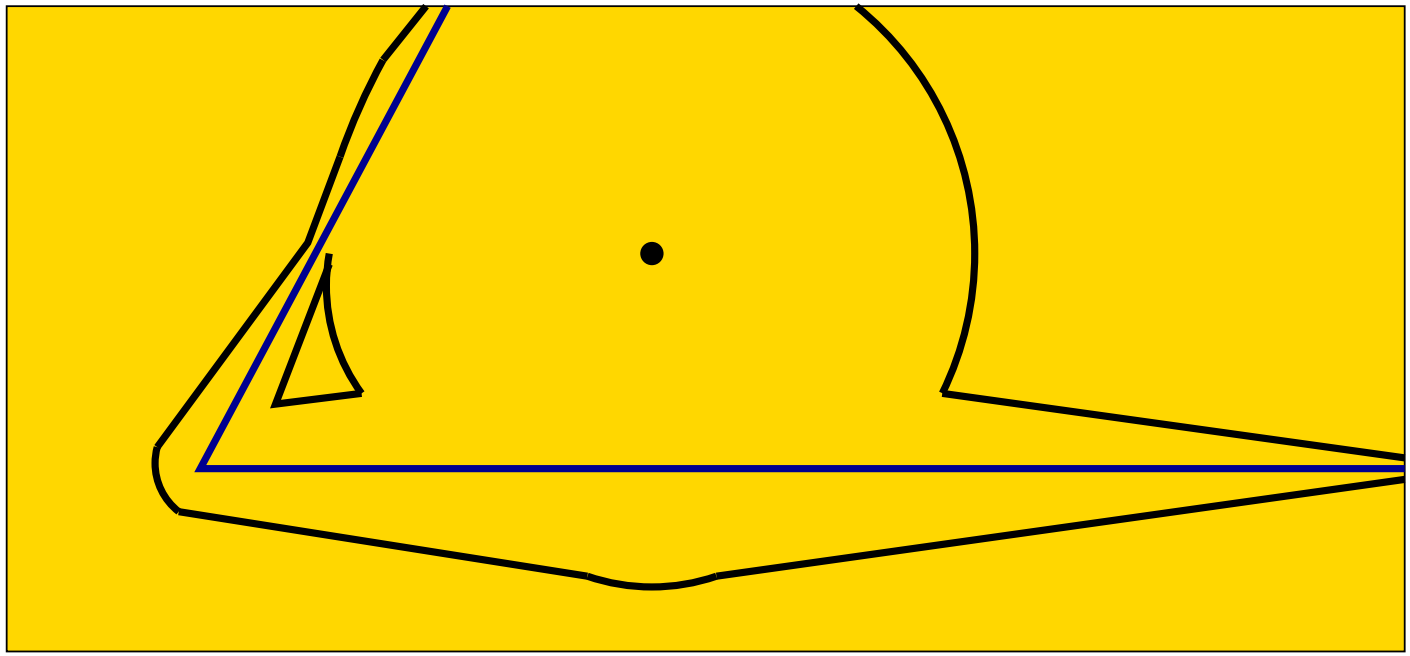

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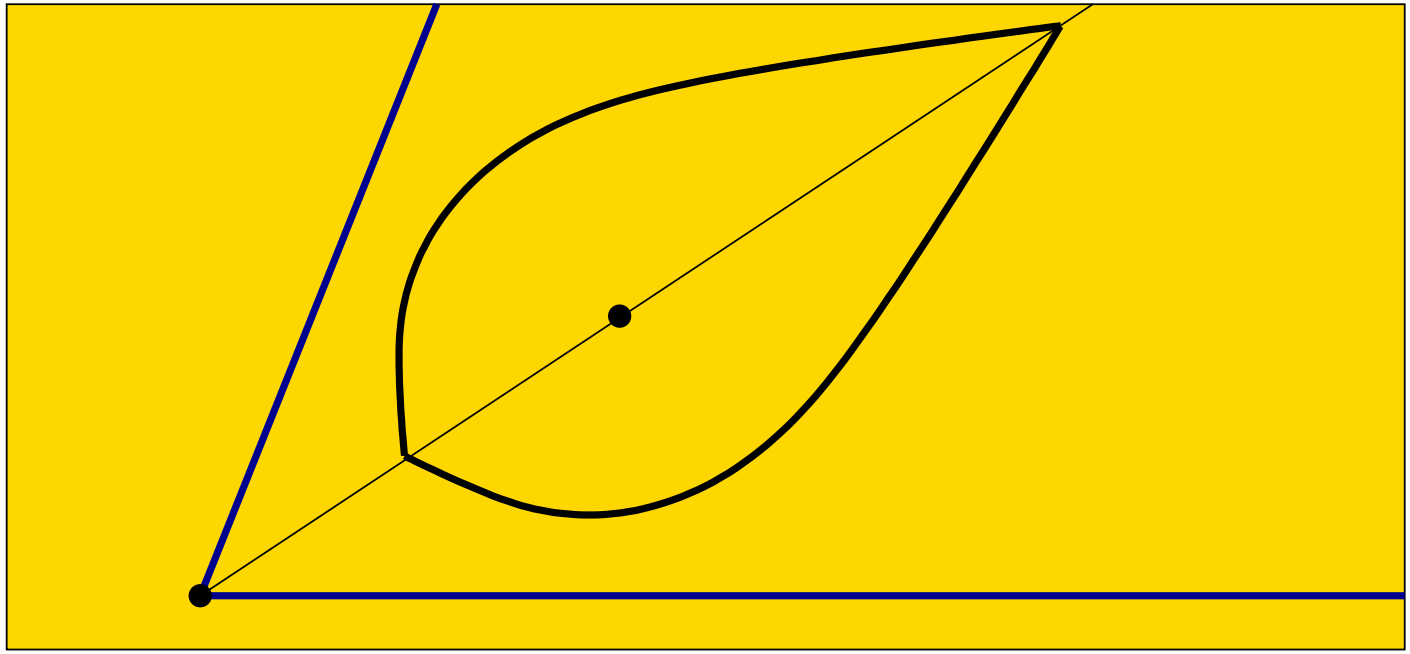
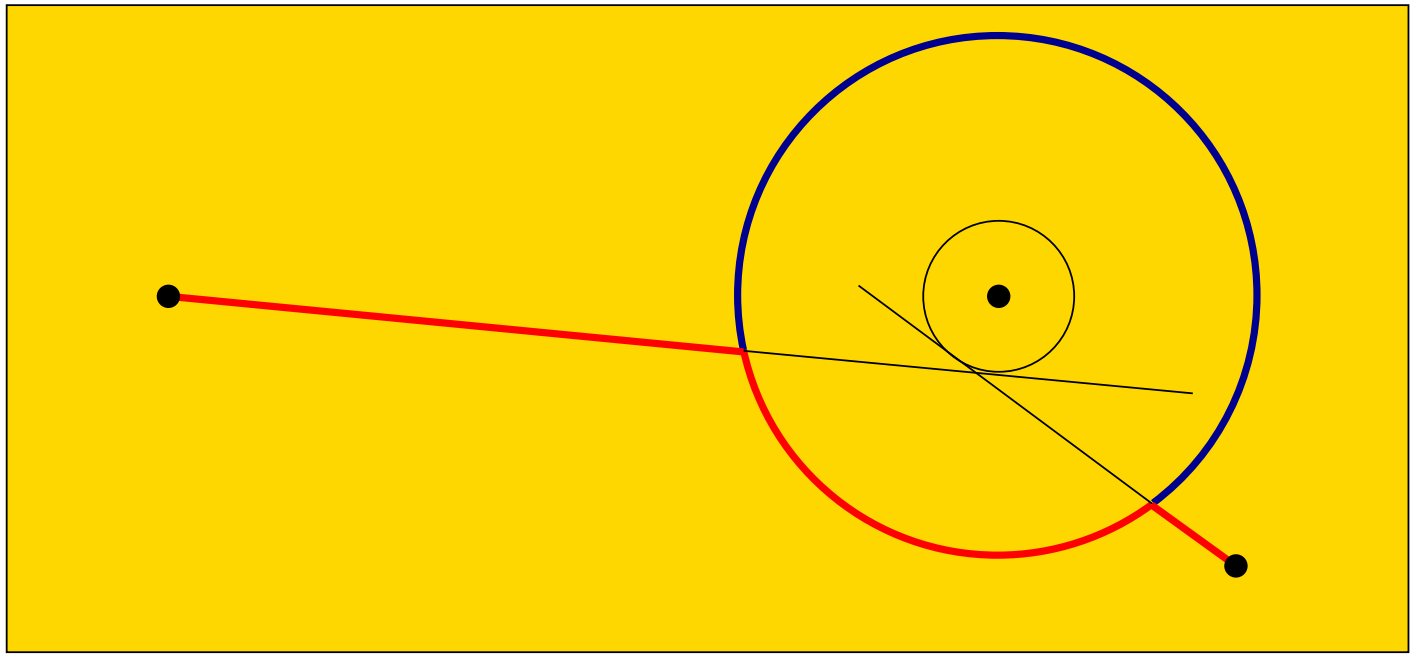


Circle Model: Description

- Transportation network is a circle C with radius r
- Speed on the plane is 1 and speed on C is $v >> 1$
- Travellers can move in the plane in any direction
- Every point in C is an access point to it

Circle Model: Description

- Shortest paths
- Chosen bisectors

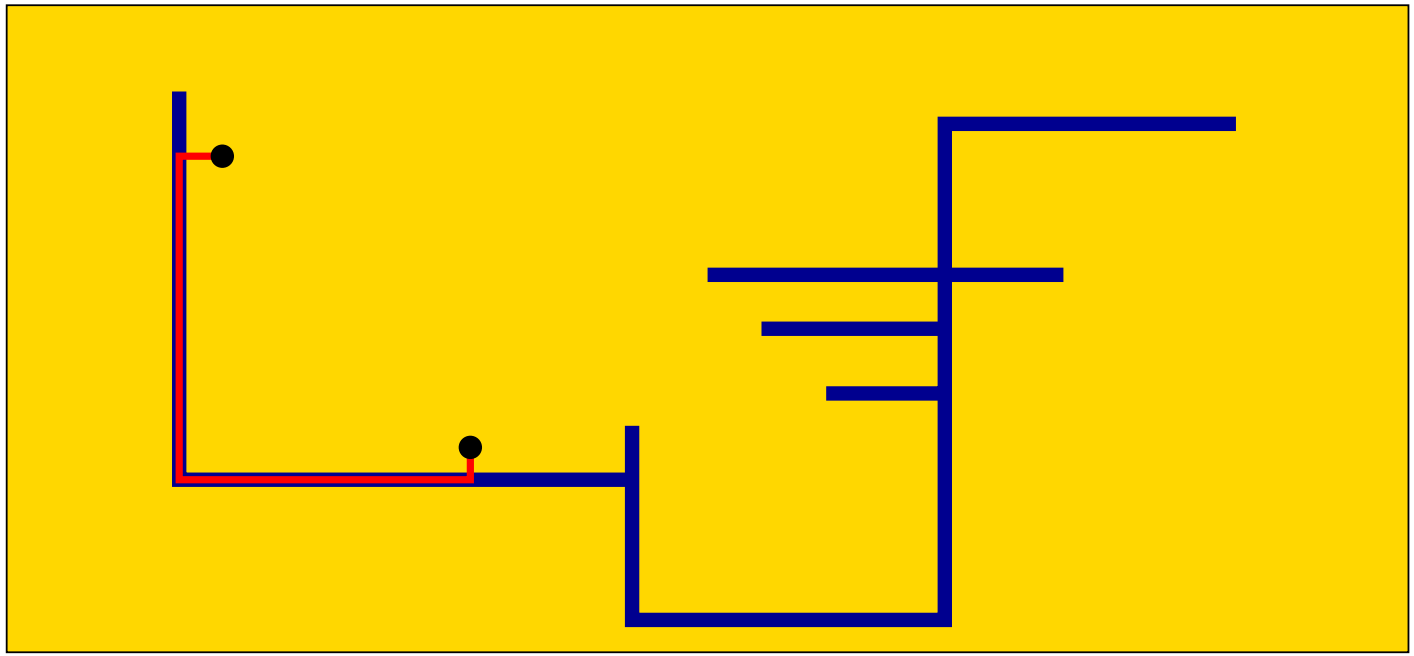


Isothetical Model: Description

- Transportation network is an isothetic planar graph C
- Speed on the plane is 1 and speed on C is $v >> 1$
- Travellers can move in the plane in isothetic directions
- Every point in C is an access point to it

Isothetical Model: Description

- **Shortest paths**
- Time circles and straight skeletons



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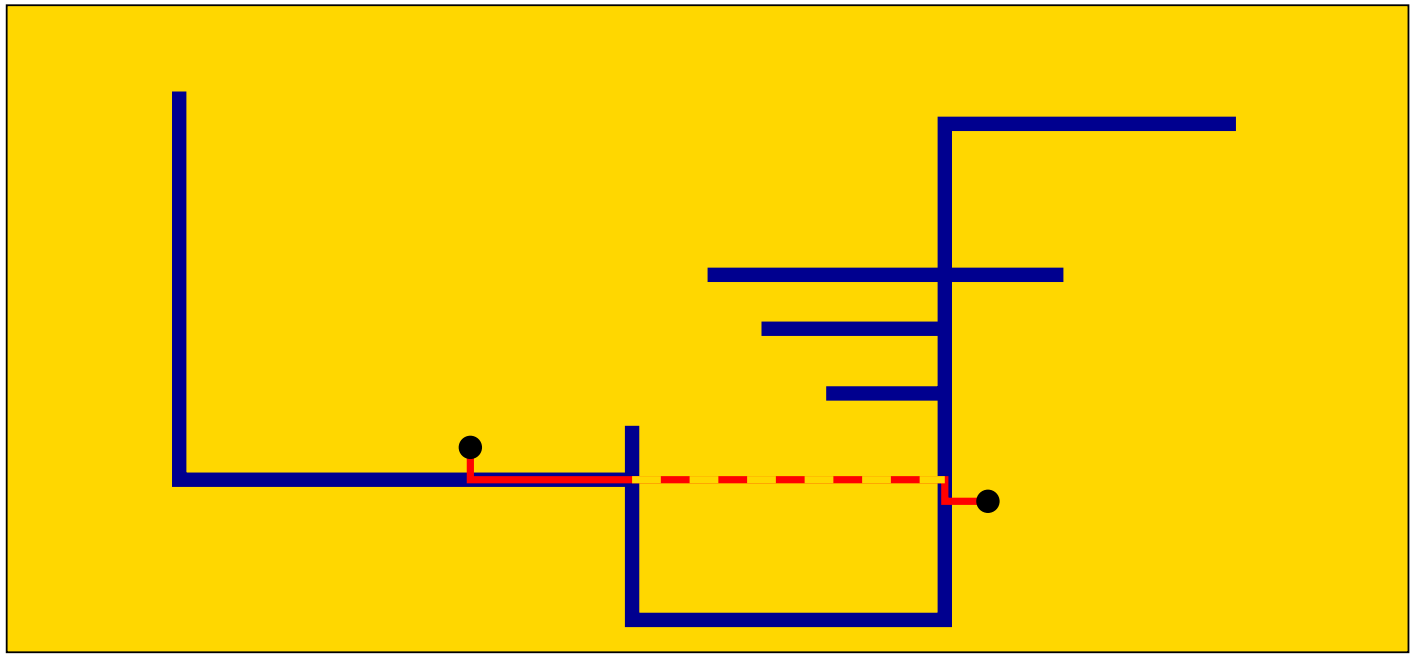

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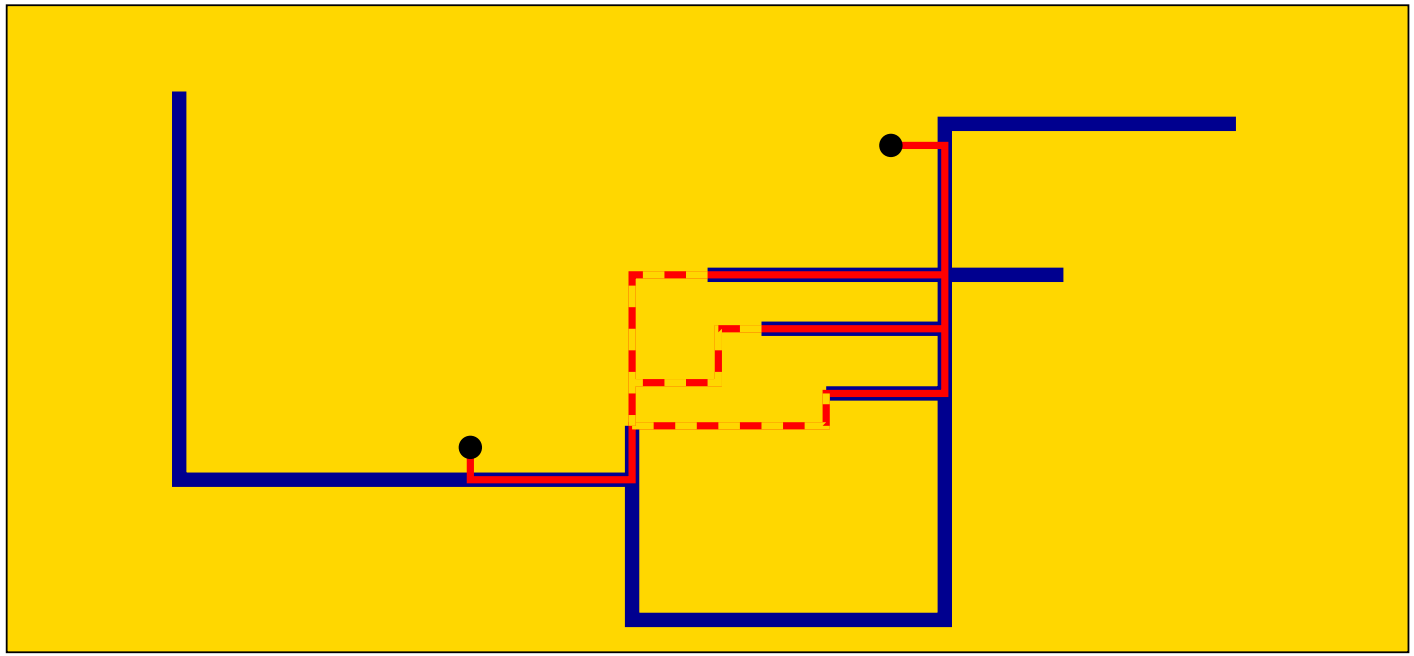

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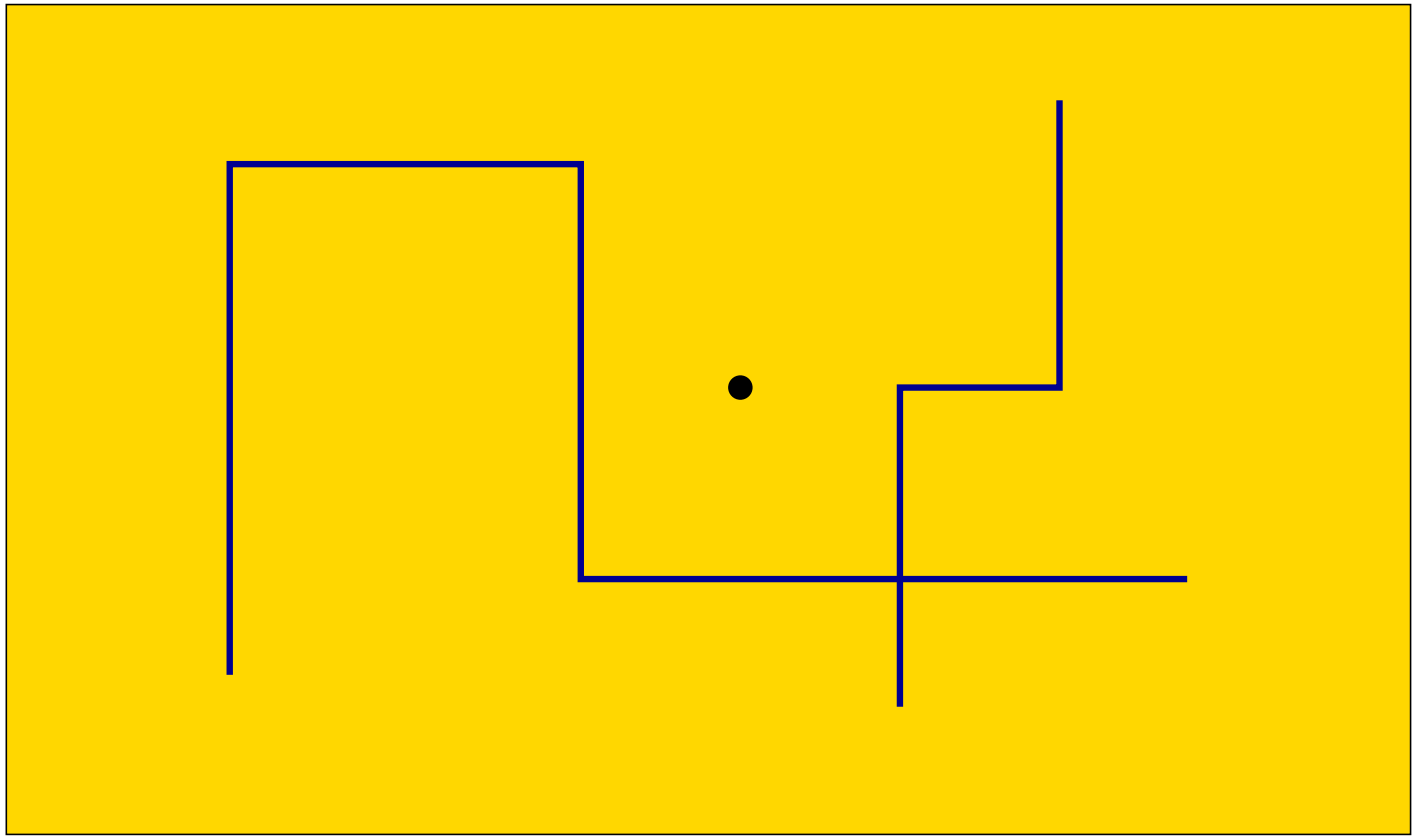

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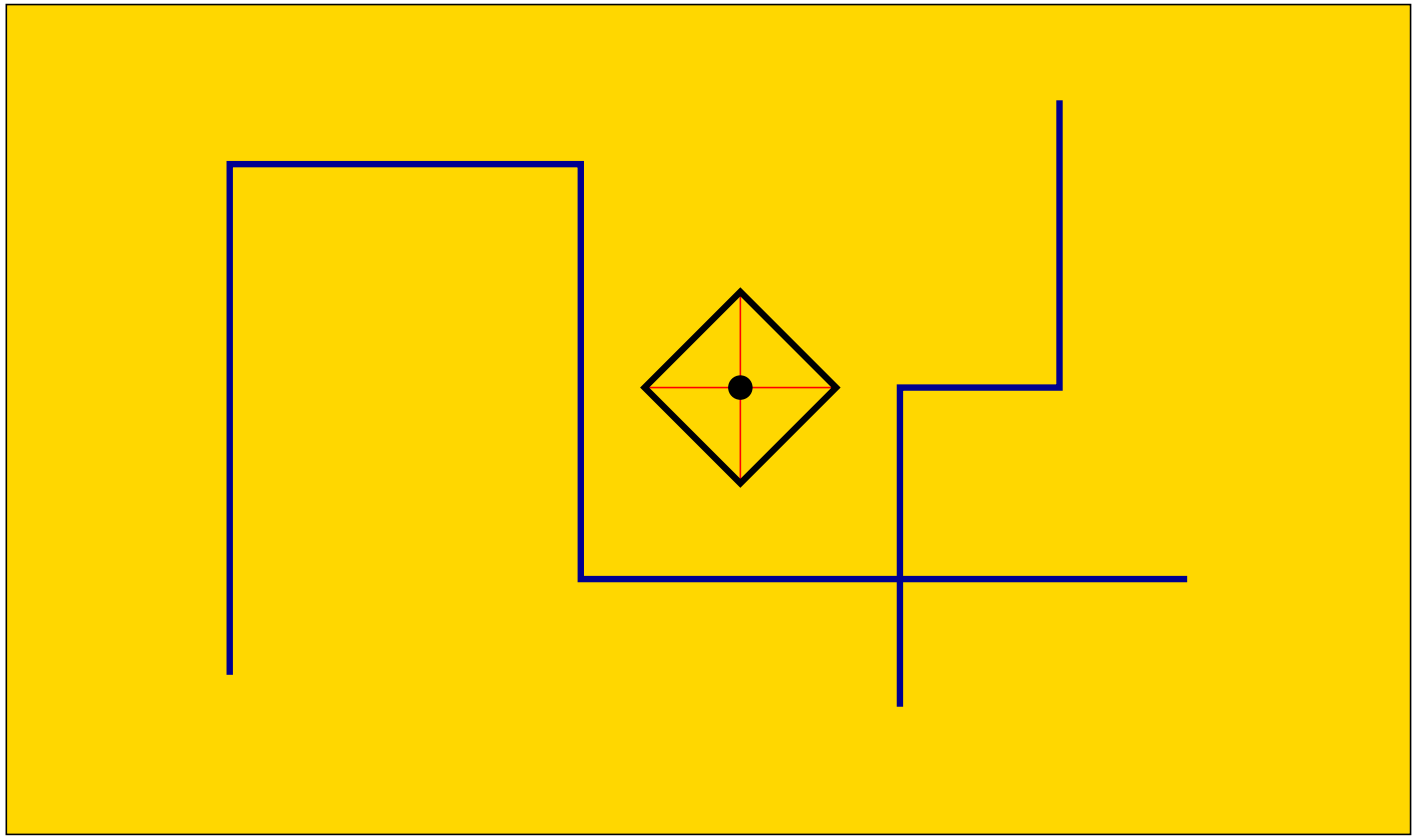

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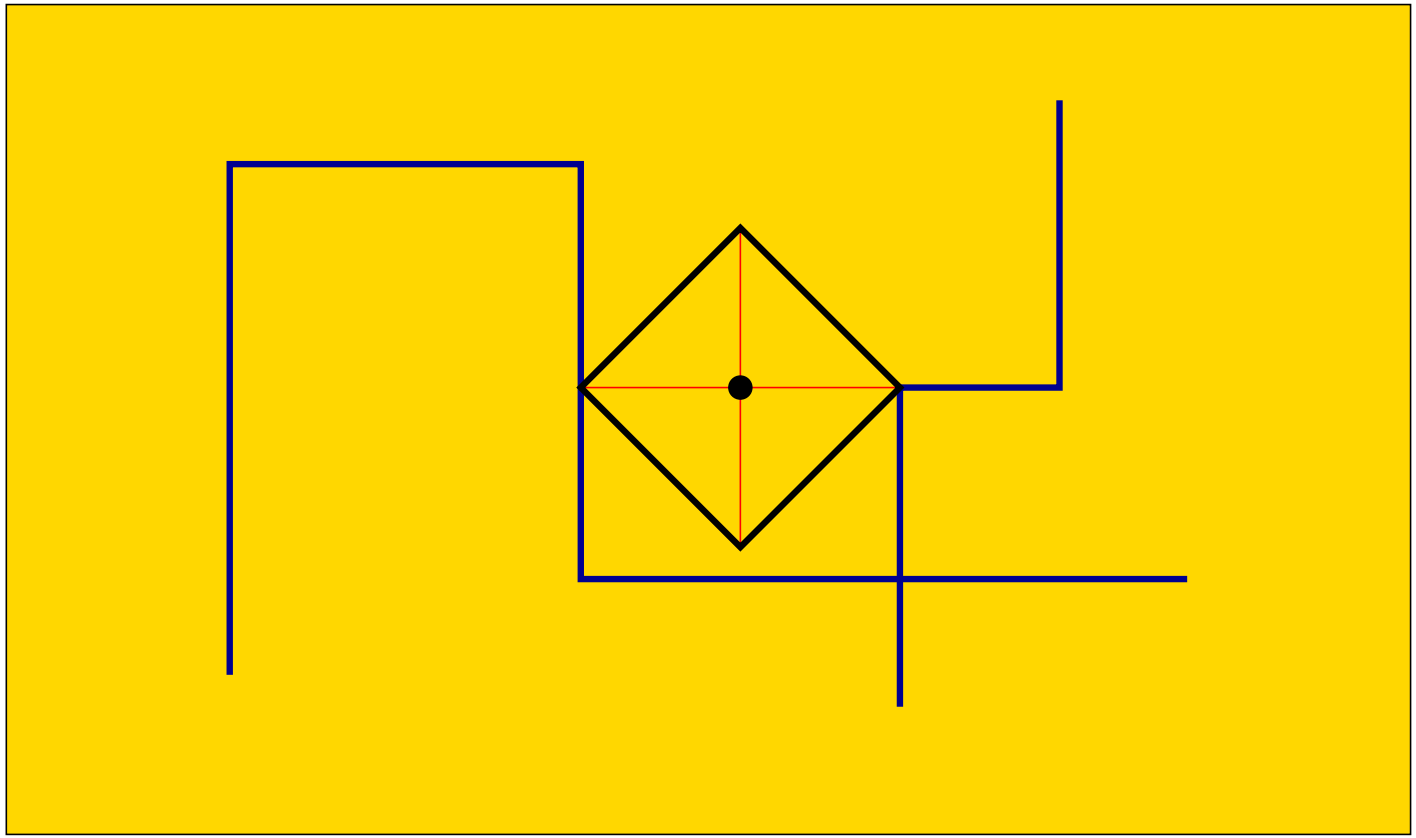

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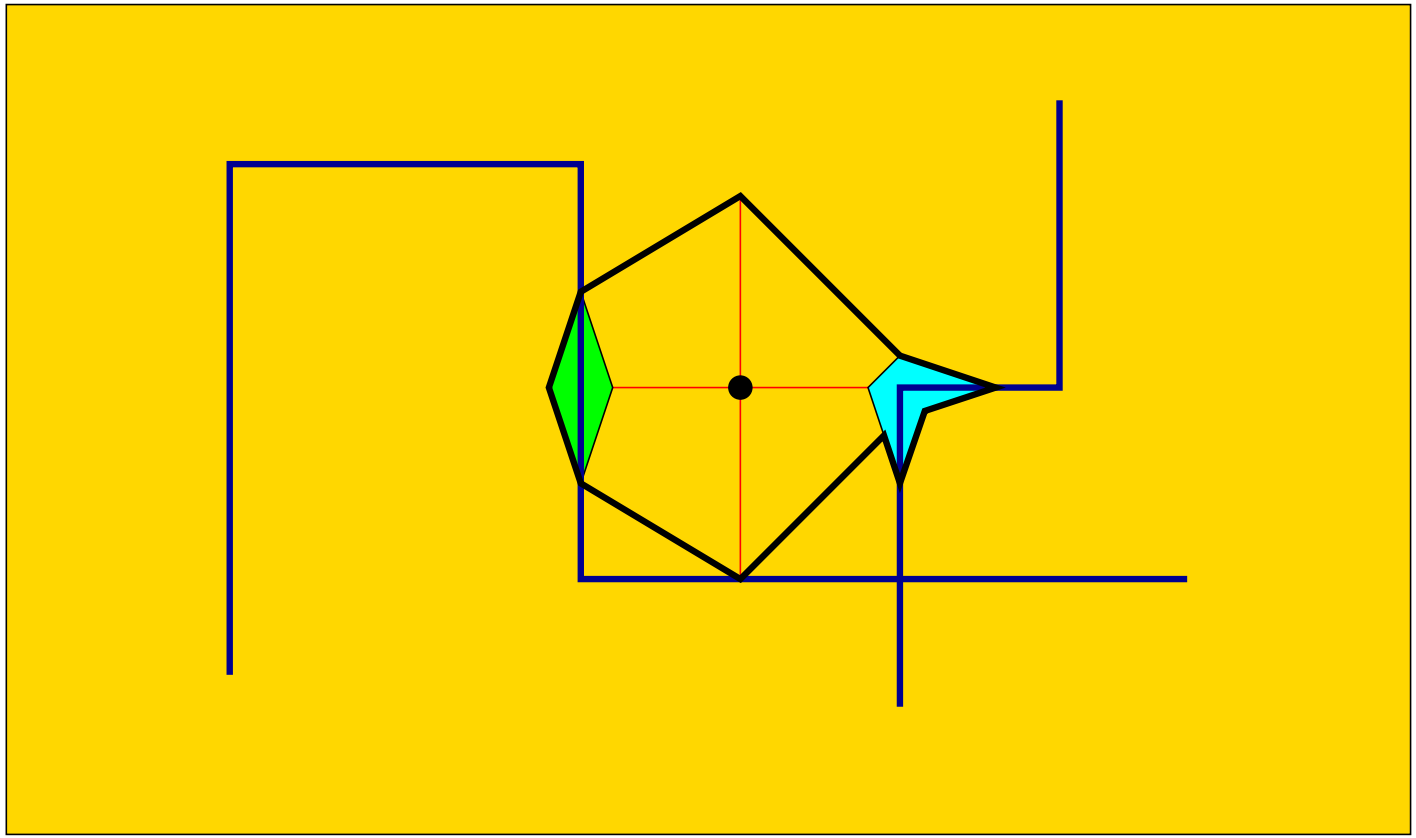

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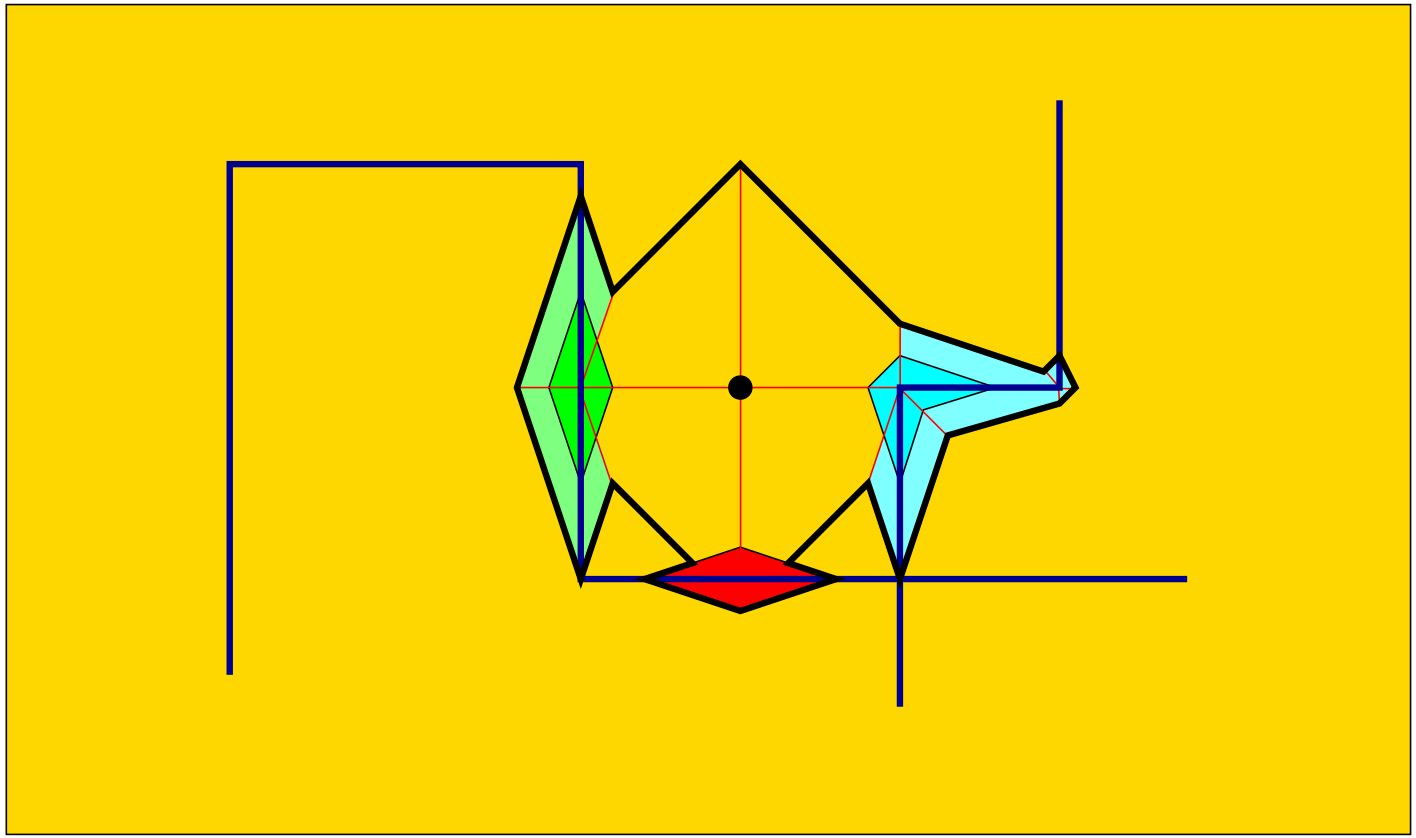

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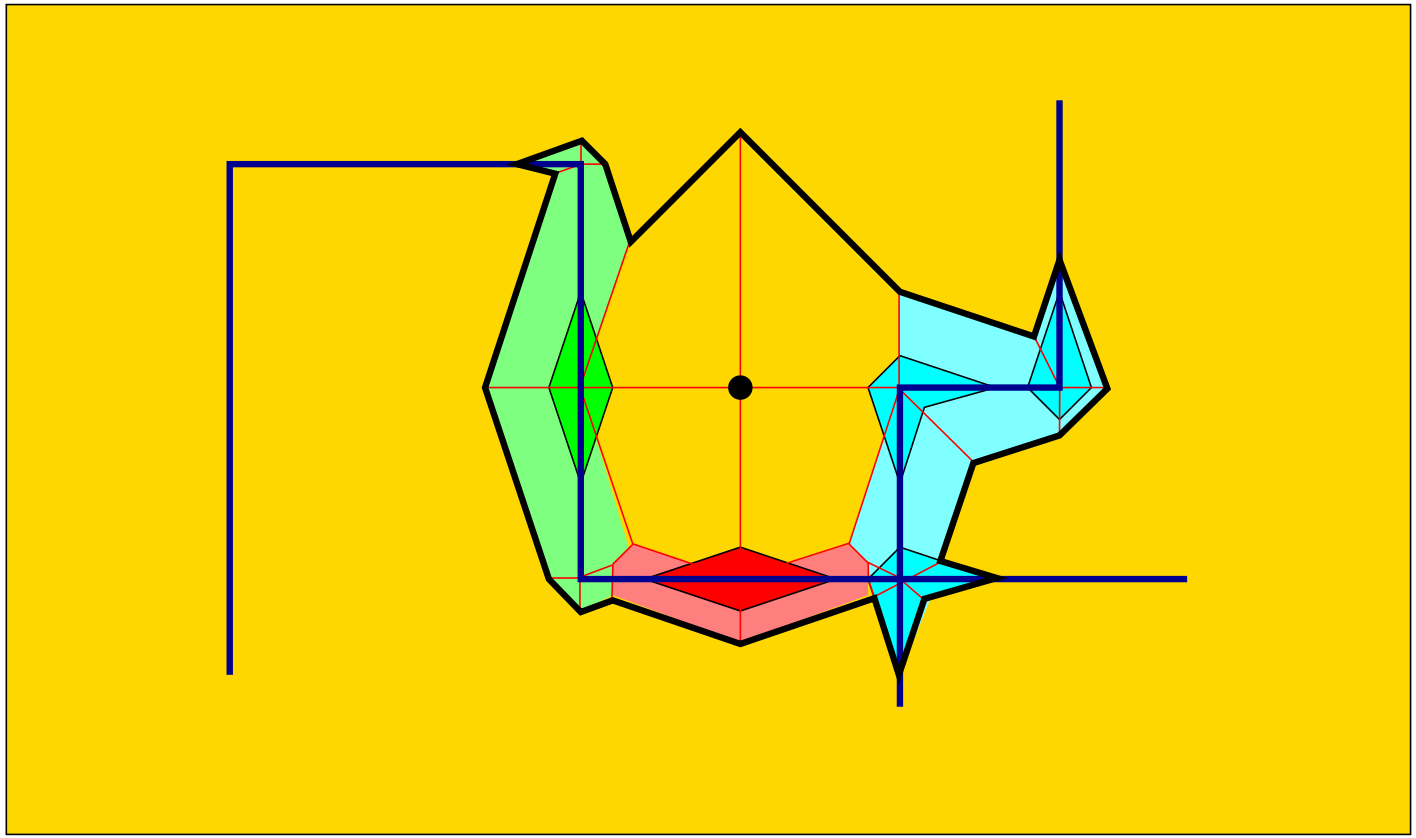

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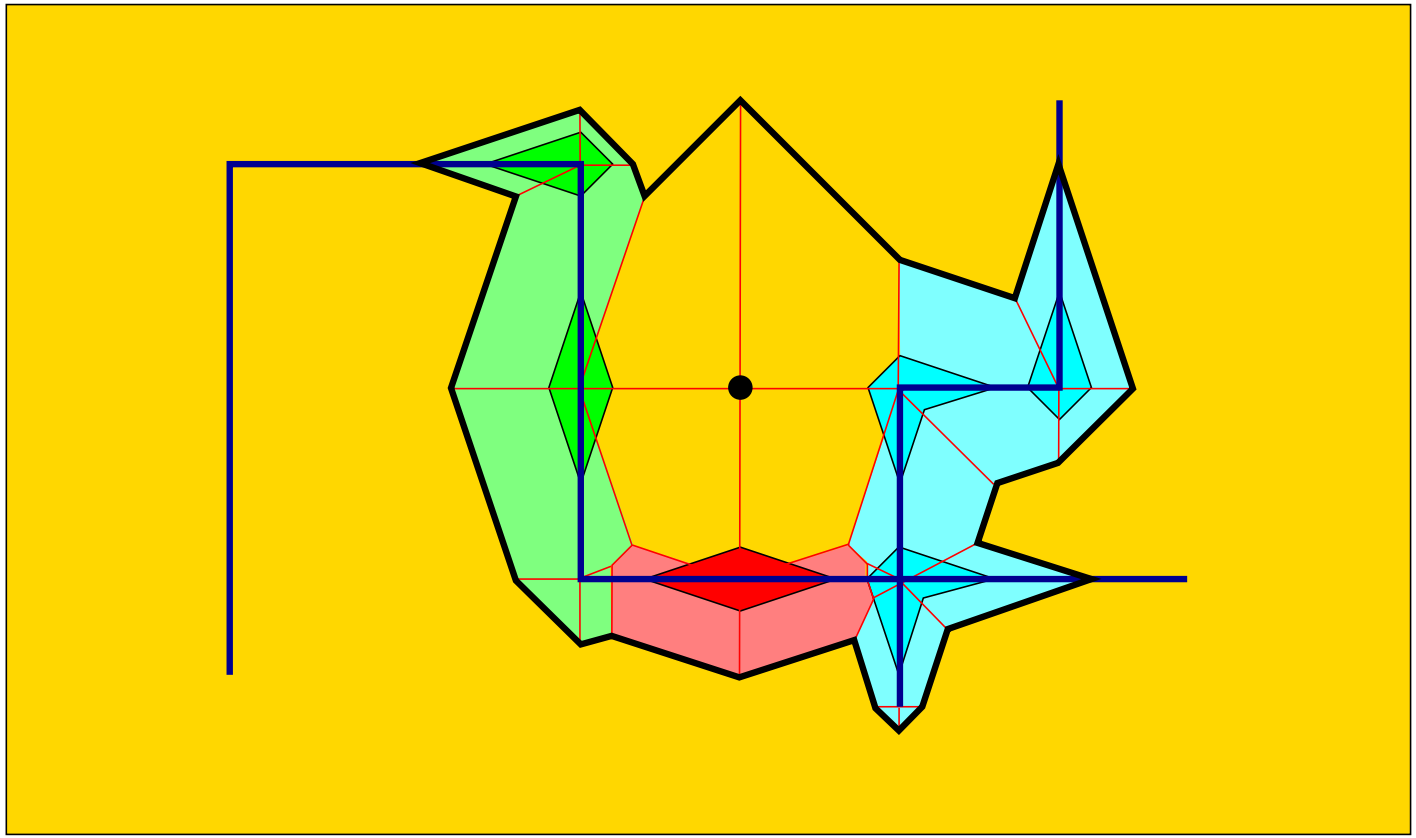

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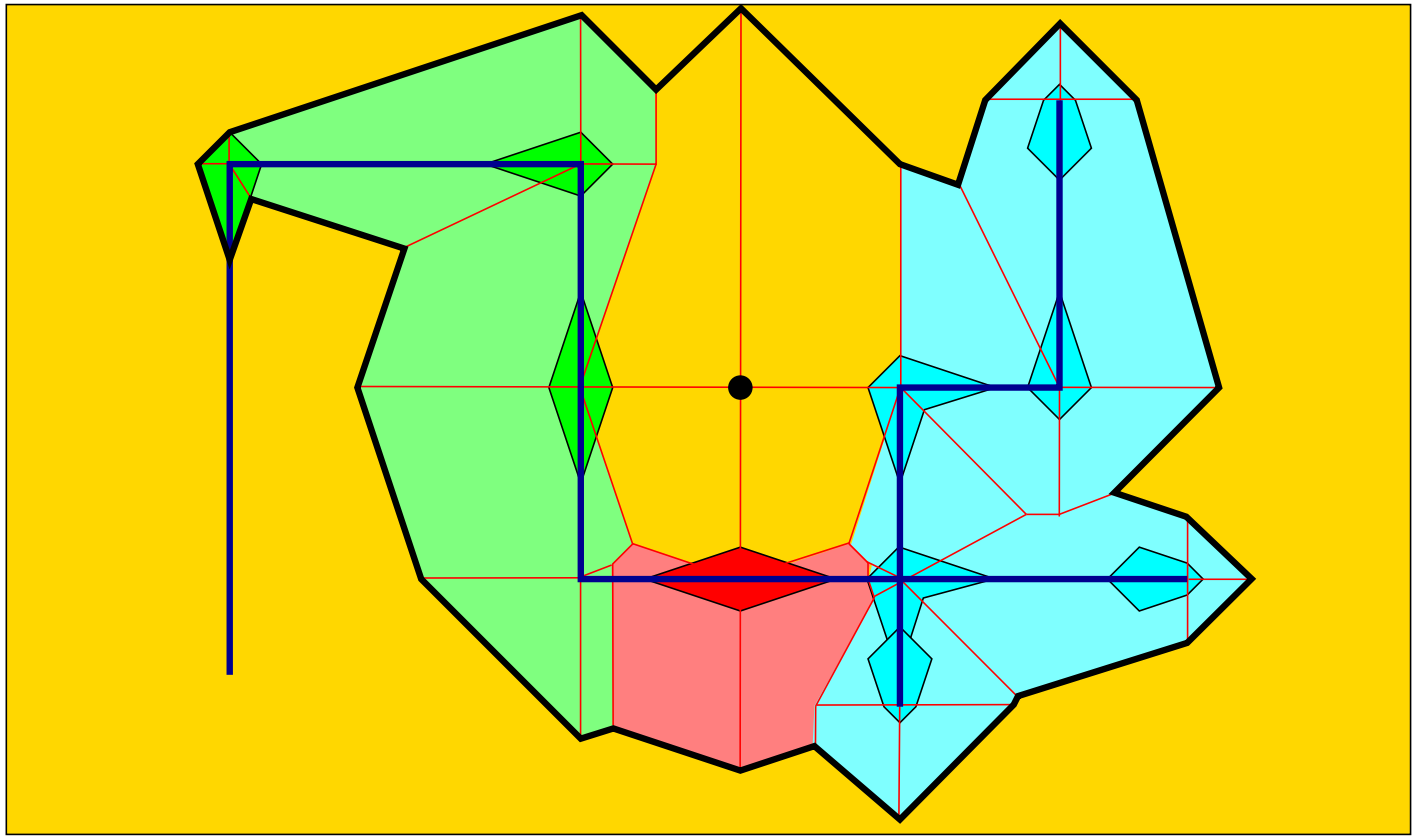

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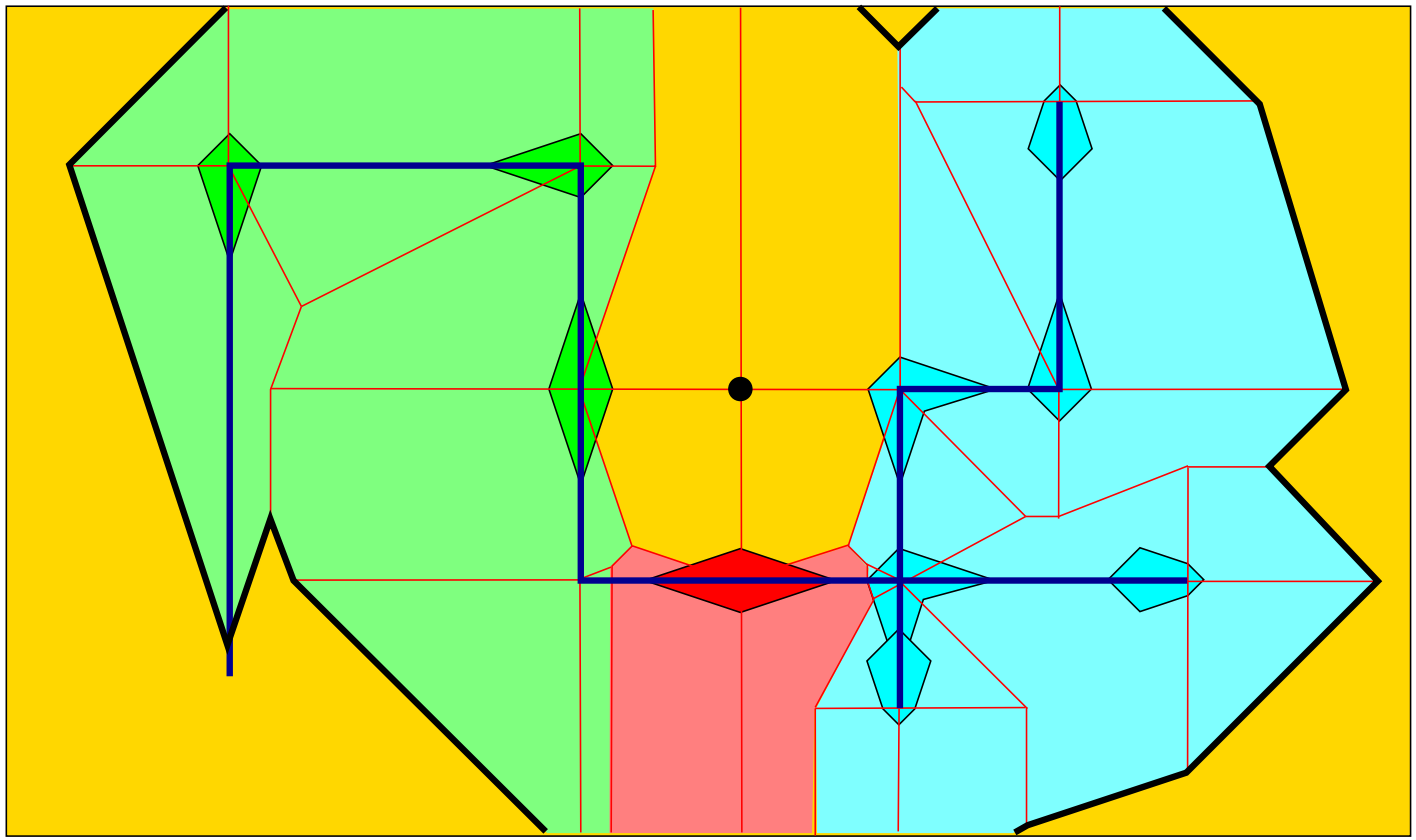

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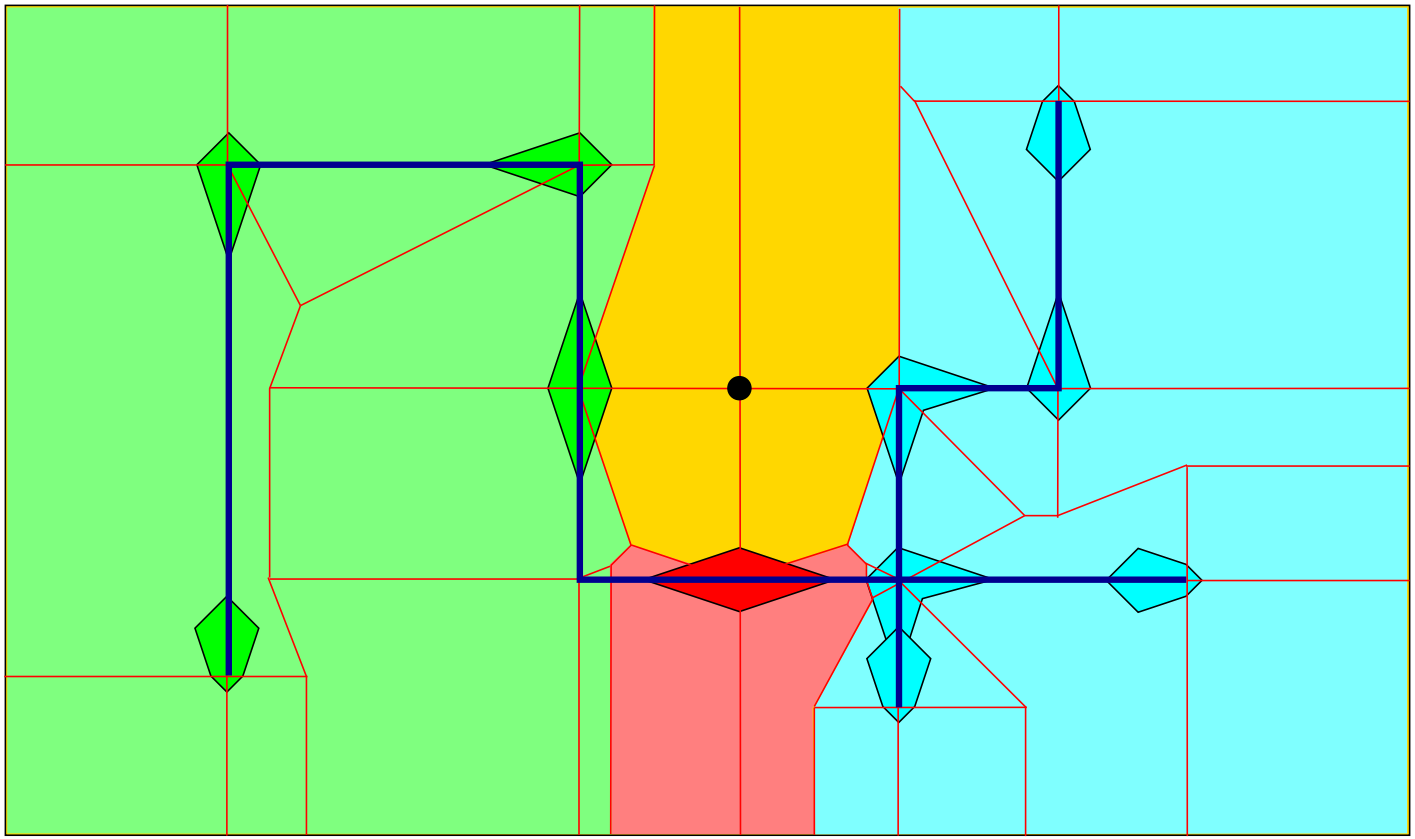

Isothetical Model: City Voronoi diagram

- **Theorem:** The straight skeleton of a set F of pairwise non-piercing figures is an abstract Voronoi diagram with $SK(\{f,g\})_{\{f,g\}} \subset F$ as its bisector system

- **Corollary:** The time Voronoi diagram for a set of n points under the isothetical model can be computed in $O(n \log n)$ time and linear space

Discrete Model: Description

- Transportation network is a directed positively weighted planar graph G
- Speed on the plane is 1 and speed on G is on edge weights
- Travellers can move in the plane in any direction
- Only the nodes in G are access points to the network

Discrete Model: Time Voronoi diagram

- **Theorem:** The time Voronoi diagram for a set of n points with a graph of c nodes can be expressed in terms of an additively weighted Voronoi diagram in $O((n+c)\log n + c^2 \log c)$ time and $O(n+c^2)$ space complexity.

- **Corollary:** The time Voronoi diagram for a set of n points with a graph of c nodes can be computed in $O((n+c)\log(n+c))$, after preprocessing.

Conclusions

- Simple network models present challenging theoretical problems.
- Is it possible to generalize the idea of the *figures* in order to decompose closed bisectors in the Wedge Model and other distances?
- Are approximate algorithms the only way to solve the Circle Model?