PHYS3080 – Part II
Semiconductors 4

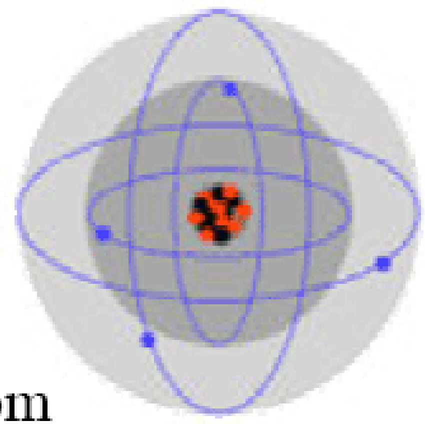
Dr Neil Curson

Email: neil@phys.unsw.edu.au

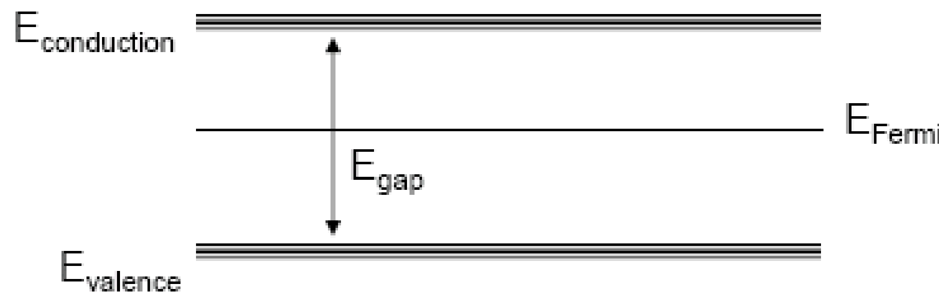
Office: HP Invent Centre, 1st floor, Newton building

Contact times: **6-7pm Wednesday**
9-10am Thursday

Electrons in solids



Atom



Semiconductor

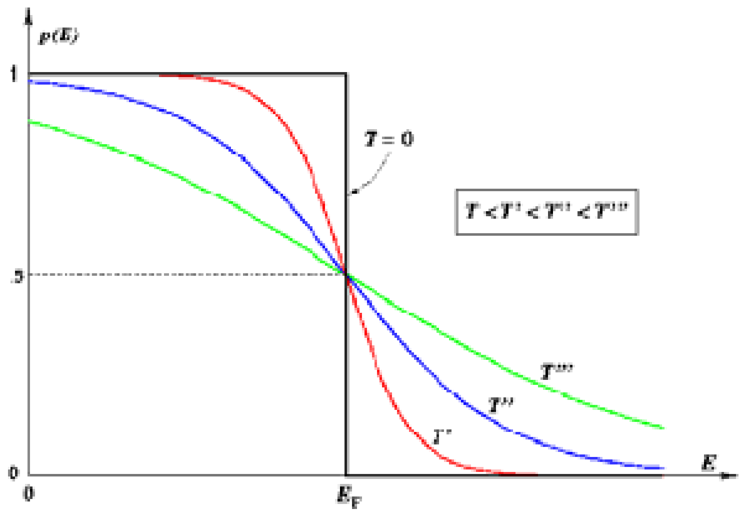
- In an atom, electrons orbit in their shell, at a given energy.
- In a crystal, many electrons occupy a small energy band. There is a width to the energy band, which is why Pauli Exclusion is not violated.
- Within the band, electrons can move easily if there are available states, because the difference in energy is tiny.
- Between bands, electrons must get energy from another source, because the band gap can be significant.

Fermi energy

- The highest energy an electron reached if you were to fill the solid with the intrinsic number of electrons at absolute zero. (No added thermal energy)
- Meaningful! There is a sea of electrons sitting beneath this energy.
  - If you bring two solids together with different Fermi energies, the electrons will move around to reach an equilibrium.

    e.g. pn junction
  - If you try to put a lower energy electron into a solid (at absolute zero) with a higher Fermi energy, it won't fit. It cannot be done due to Pauli Exclusion.
- If the highest energy electron exactly fills a band, the Fermi Energy is near the center of the bands.

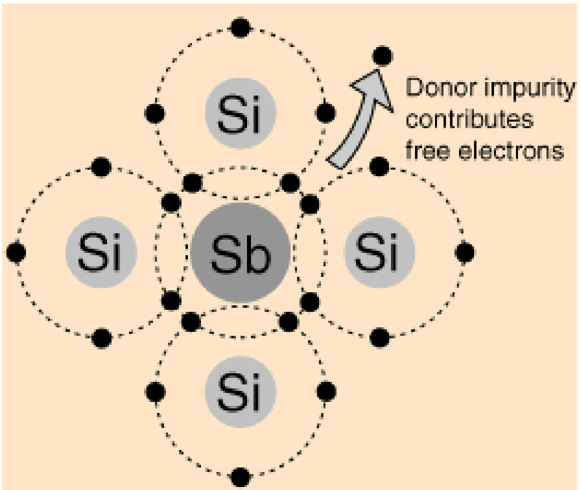
Above 0 K: Fermi-Dirac statistics

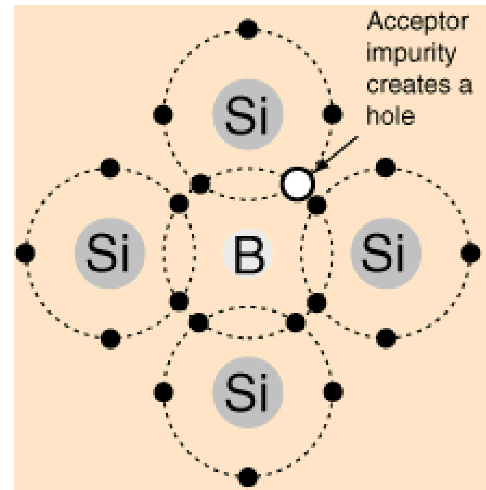


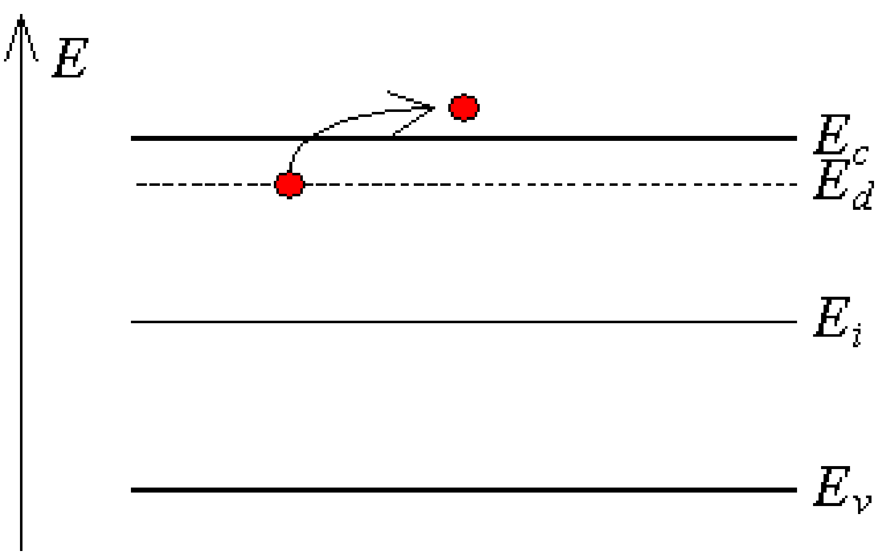
$$f(E) = \frac{1}{1 + e^{\frac{E - E_F}{kT}}}$$

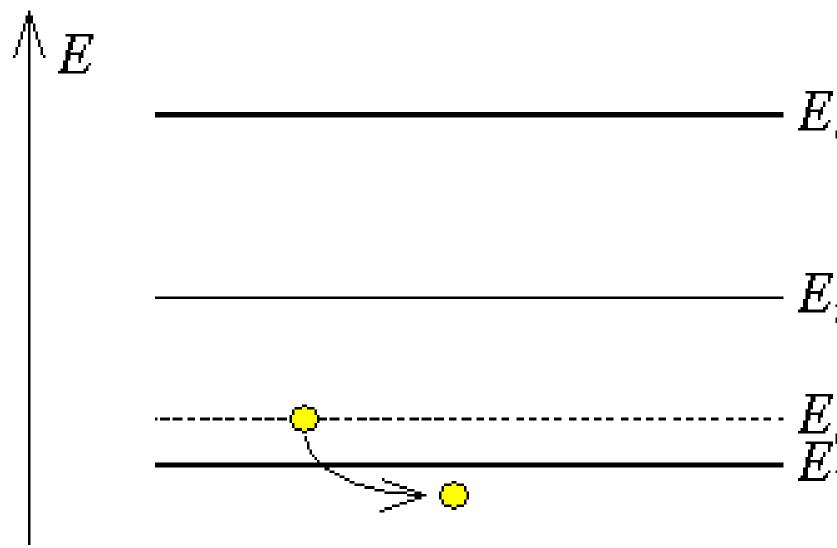
- Fermi Energy: The energy state whose probability of being occupied is exactly 1/2 .
- Electrons obey Fermi-Dirac statistics, which describe the probability of an electron being present in an allowed energy state.
- Note that if there are no states at a given energy (i.e., in the band gap) there will be no electrons, even if there is finite probability.

Doped (extrinsic) semiconductors

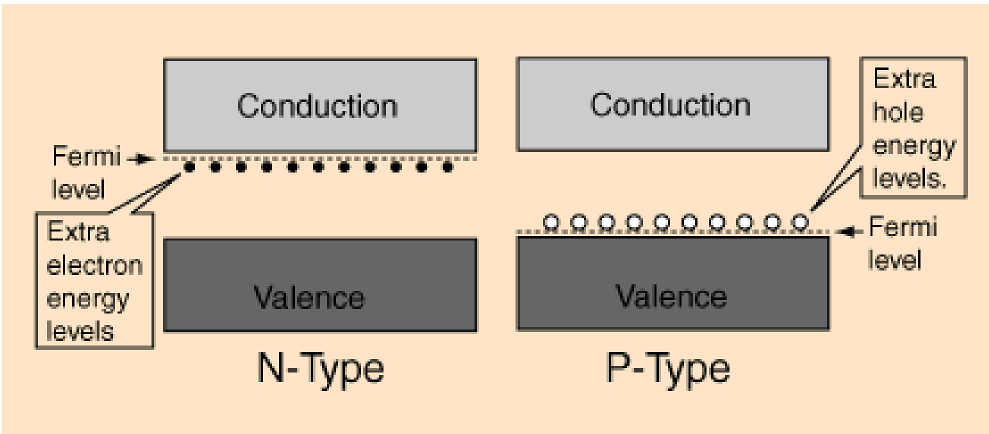


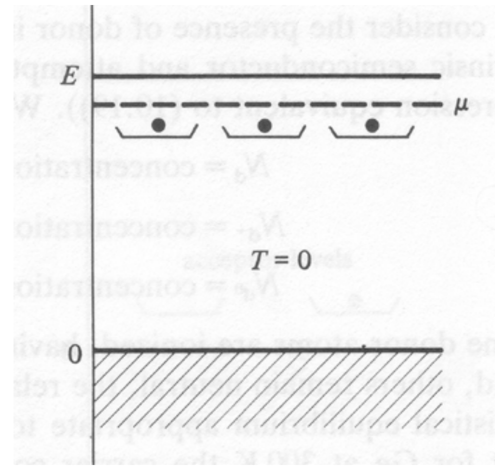


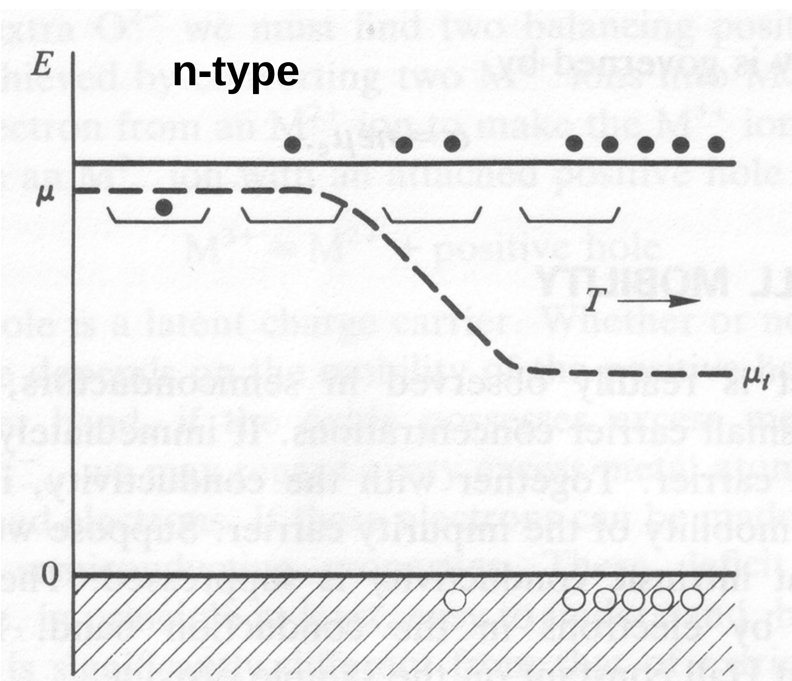


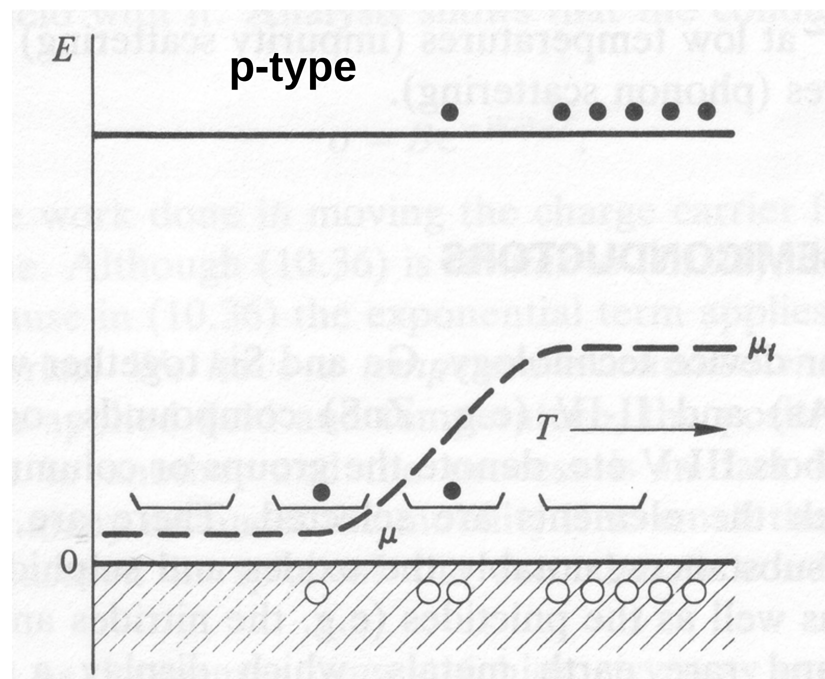


Fermi level positions

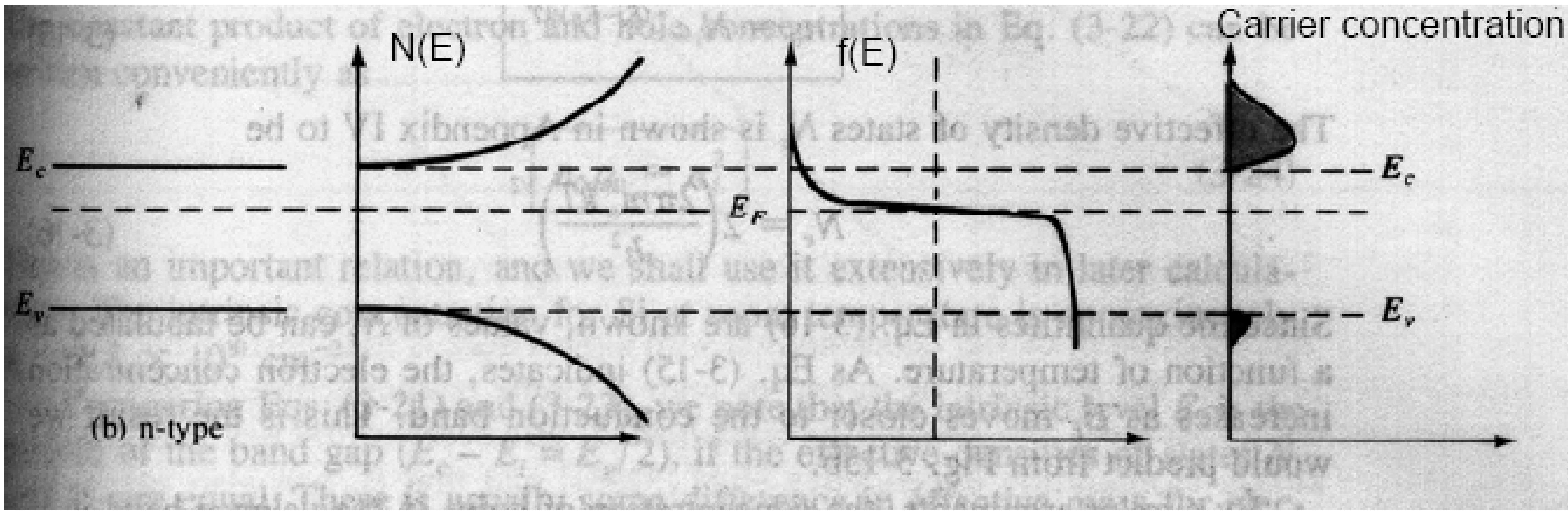








Equilibrium concentration: electrons



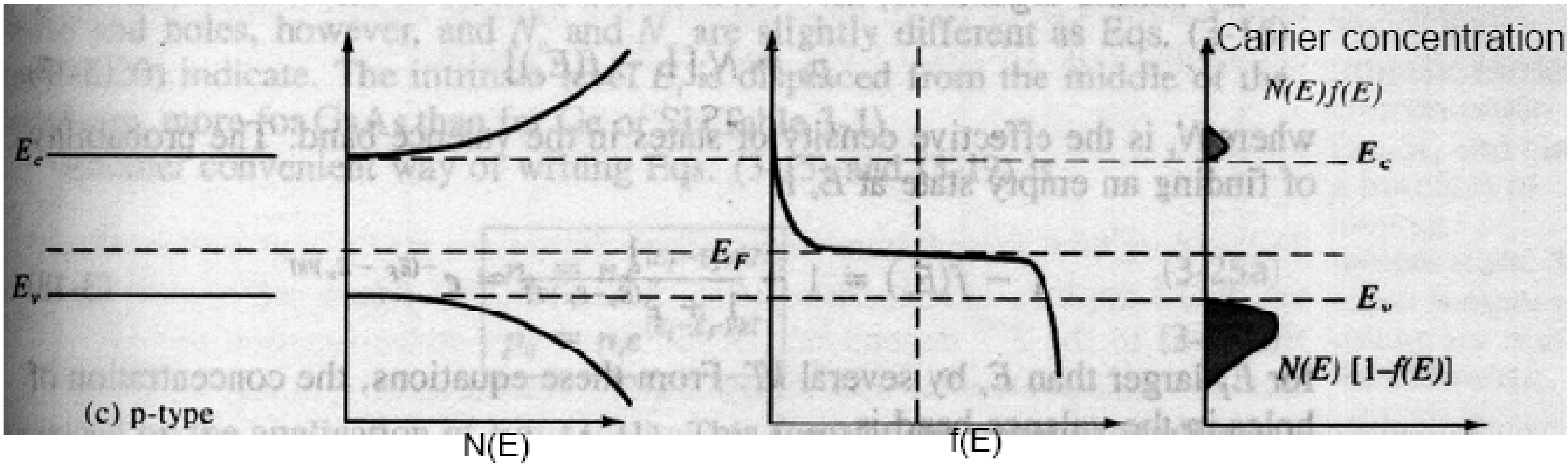
$$n_0 = \int_{E_c}^{\infty} f(E)N(E)\,dE = N_c f(E_c) = \frac{N_c}{1+e^{(E_c-E_F)/kT}} \approx N_c e^{-(E_c-E_F)/kT}$$

$n_0 =$ equilibrium electron carrier concentration

$N(E) =$ density of states

$N_c = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2} =$ effective density of states

Equilibrium concentration: holes



$$p_0 = \int_{-\infty}^{E_v} (1 - f(E))\, N(E)\, dE = N_v\,(1 - f(E_c)) = \frac{N_v}{1 + e^{(E_F - E_v)/kT}} \approx N_v e^{-(E_F - E_v)/kT}$$

$p_0 =$ equilibrium hole carrier concentration

$N(E) =$ density of states

$N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2} =$ effective density of states

Intrinsic semiconductors

- In intrinsic semiconductors (no doping) the electron and hole concentrations are equal because carriers are created in pairs

$$n_i = N_c e^{-(E_c - E_i)/kT} = p_i = N_v e^{-(E_i - E_v)/kT}$$

$$n_0 p_0 = \left(N_c e^{-(E_c - E_F)/kT}\right)\left(N_v e^{-(E_F - E_v)/kT}\right)$$

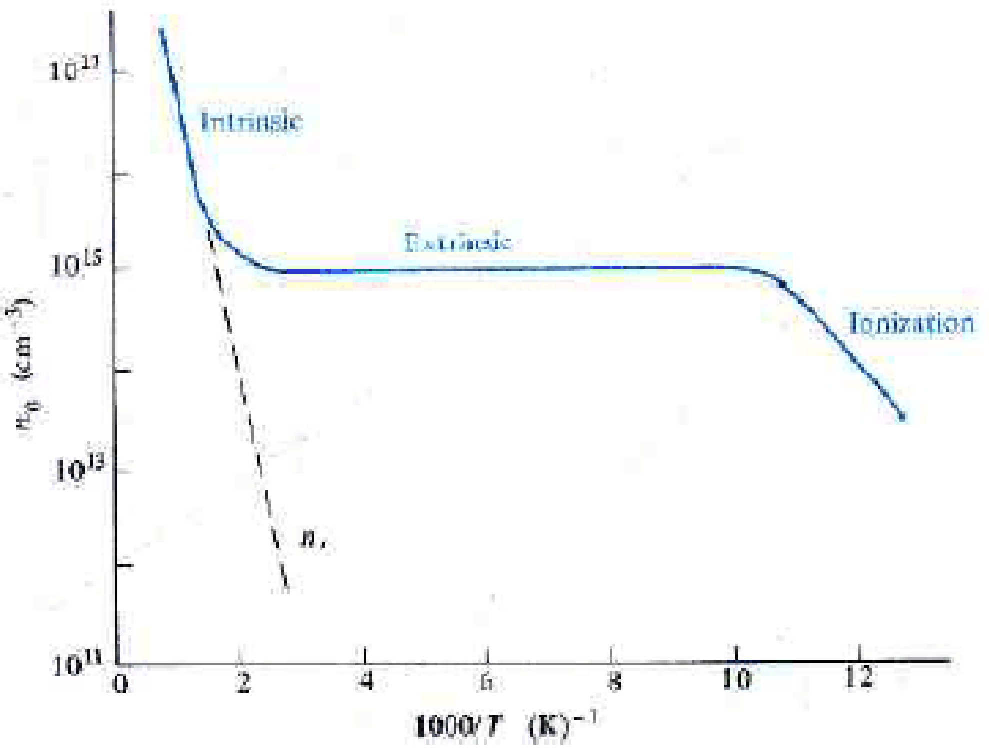
$$= N_c N_v e^{-(E_c - E_v)/kT} = N_c N_v e^{-E_g/kT} = n_i p_i = n_i^2$$

- This allows us to write

$$n_0 = n_i e^{(E_F - E_i)/kT} \qquad p_0 = n_i e^{(E_i - E_F)/kT}$$

  - As the Fermi level moves closer to the conduction [valence] band, the n_0 [p_0] increases exponentially

Temperature dependence of carrier concentrations



$$n_0 = \left[2\left(\frac{2\pi kT}{h^2}\right)^{3/2}\left(m_n^* m_p^*\right)^{3/4} e^{-E_g/2kT}\right] e^{(E_F-E_i)/kT}$$

- The intrinsic concentration depends exponentially on temperature. The T^3 dependence is negligible.
- Ionization: only a few donors [acceptors] are ionized.
- Extrinsic: All donors [acceptors] are ionized
- Intrinisic: As the temperaure increases past the point where it is high enough to excite carriers across the full band gap, intrinsic carriers eventually contribute more.
- At room temp (300K), the intrinsic carrier concentration of silicon is:

$$n_i = 1.5 \times 10^{10}/\text{cm}^3$$

Current flow

- There are two mechanisms by which mobile carriers move in semiconductors – resulting in current flow
  - Drift
    - Carrier movement is induced by a force of some type
  - Diffusion
    - Carriers move (diffuse) from a place of higher concentration to a place of lower concentration

$$J_n(x) = \underbrace{qn\mu_n\mathcal{E}}_{\text{drift}} + \underbrace{qD_n\frac{dn(x)}{dx}}_{\text{diffusion}}$$

Drude model of conductivity

- Electrons are assumed to move in a direct path, free of interactions with the lattice or other electrons, until it collides.
- This collision abruptly alters its velocity and momentum.
- The probabilty of a collision occuring in time *dt* is simply dt/τ, where τ is the mean free time. τ is the average amount of time it takes for an electron to collide.

$$J = -qnv_{avg} = qn\mu_n\mathcal{E}$$

- The current is the charge*number of electrons*area*velocity in a unit of time. For j = current density, divide by the area. The drift velocity (v_d) is a function of charge mobility (μ_n) and electric field (*E*).
- At equilibrium, there is no net motion of charge, v_{avg} = 0.
- With an applied electric field, there is a net drift of electrons [holes] against [with] the electric field resulting in an average velocity.
- This model allows us to apply Newton's equations, but with an effective mass. The effective mass takes the interactions with the rest of the solid into account.

Drude model of conductivity

$$v = v_0 + at \quad F = qE = ma \quad a = \frac{qE}{m}$$

- Consider an electron just after a collision. The velocity it acquires before the next collision will be acceleration*time

$$v_{avg} = -\frac{qE\tau}{m^*} \qquad J = \sigma E = \left(\frac{nq^2\tau}{m^*}\right)E \qquad \sigma = \frac{nq^2\tau}{m^*}$$

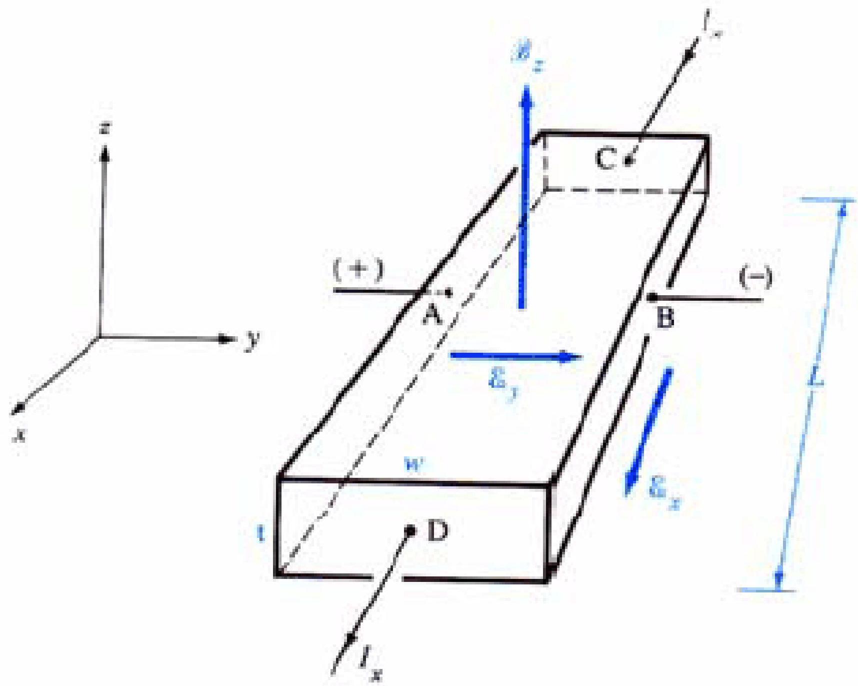
- We want the average velocity of all the electrons, which can be obtained by simply averaging the time, which we already know is τ.
- We can also write this in terms of mobility:

$$\sigma = qn\mu \qquad \mu = \frac{qt}{m^*} = -\frac{v_{avg}}{E}$$

- Taking both holes and electrons into account, we end up with the following formula for current density due to **drift**.

$$\boldsymbol{J = q\left(n\mu_n + p\mu_p\right)E}$$

Hall effect



- Moving electrons experience a force due to a perpendicular B field

$$F = q\left(\mathcal{E} + v \times B\right)$$

- An electric field develops in response to this force.
- The sign of this field perpendicular to the flow of current determines the carrier type.
- Density and mobility can also be calculated.

$$E_y = \frac{J_x B_z}{qn} \qquad \mu = \frac{1}{qn\rho}$$

$$\rho = \text{ resistivity}$$

Diffusion

- Diffusion results in a net flux of particles from the region of higher concentration to the region of lower concentration
  - This flux leads to current (movement of charged particles)
  - Magnitude of current depends on the gradient of concentration

$$J_{n,\mathrm{diffusion}}(x) = qD_n \frac{dn(x)}{dx}$$

  - D_n is the diffusivity coefficient

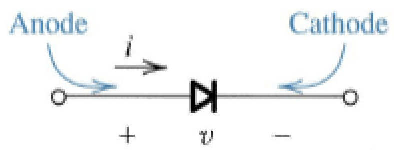
- Diffusivity is related to mobility by Einstein's relationship

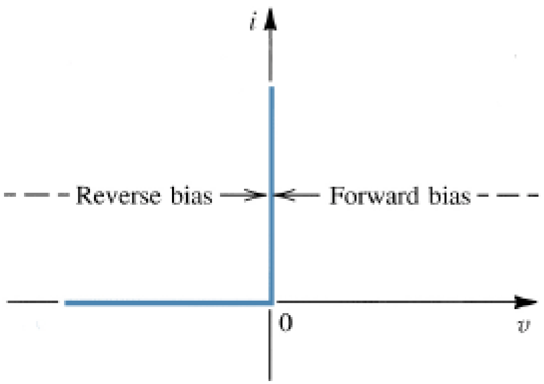
$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{q}$$

  - Typical values for Si at room temp
    - $D_n = 34\ cm^2/s$ and $D_p = 13\ cm^2/s$

pn junction diodes

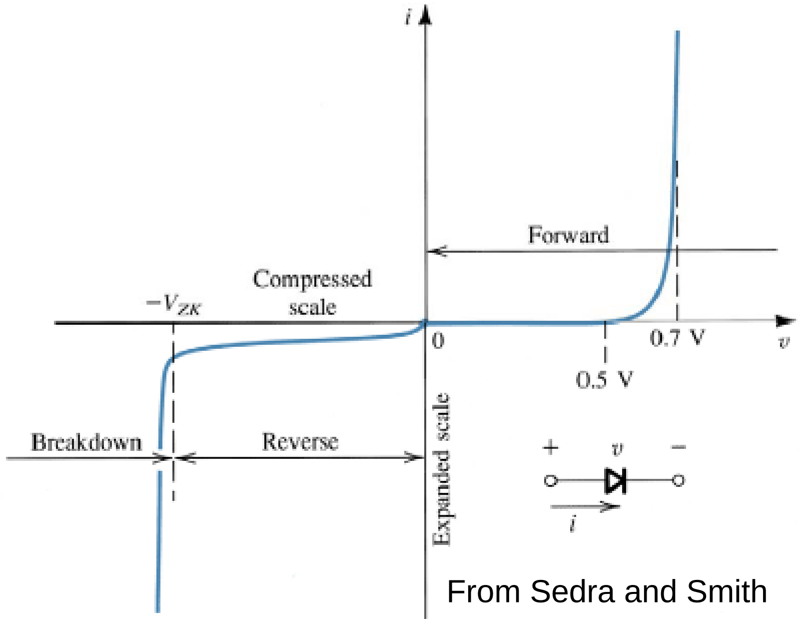
Ideal diode





Characteristics of pn junction diode

- Given a semiconductor PN junction we get a diode with the following current-voltage (IV) characteristics.
- “Turn on” voltage based on the “built-in” potential of the PN junction
  - Reverse bias breakdown voltage due to avalanche breakdown (on the order of several volts)



From Sedra and Smith

Current equations

- The forward bias current is closely approximated by

$$i = I_s\left(e^{v/nV_T} - 1\right) \text{ where } V_T = kT/q$$

where V_T is the thermal voltage (~25mV at room temp)

k = Boltzman's constant = 1.38 x 10^{-23} joules/kelvin

T = absolute temperature

q = electron charge = 1.602 x 10^{-19} coulombs

n = constant dependent on material, between 1 and 2 (we will assume n = 1)

I_S = scaled current for saturation current that is set by dimensions

  – Notice there is a strong dependence on temperature
  – We can approximate the diode equation for $i \gg I_s$

$$i \cong I_s e^{v/nV_T}$$

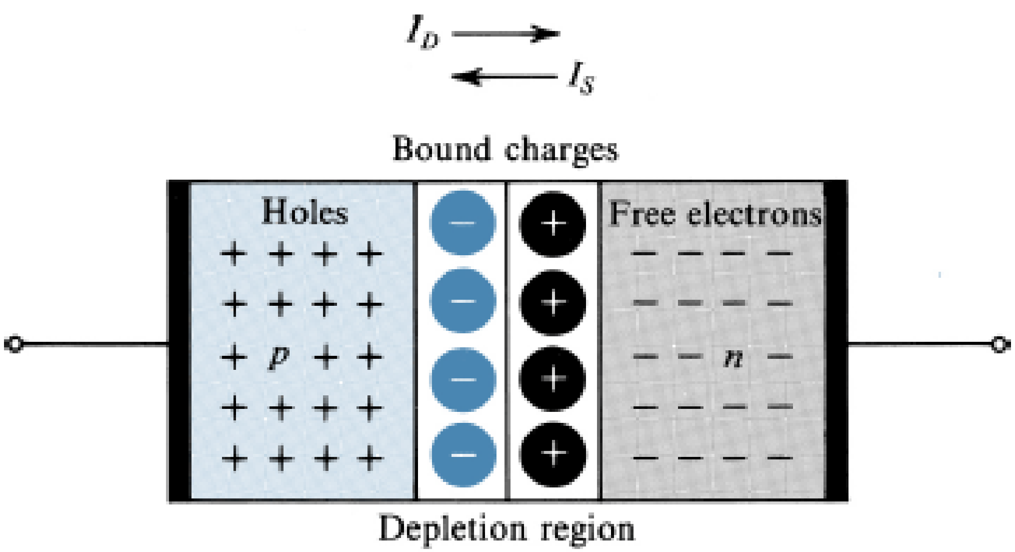
- In reverse bias (when v << 0 by at least V_T), then

$$i \cong -I_s$$

- In breakdown, reverse current increases rapidly… a vertical line

Mobile carriers

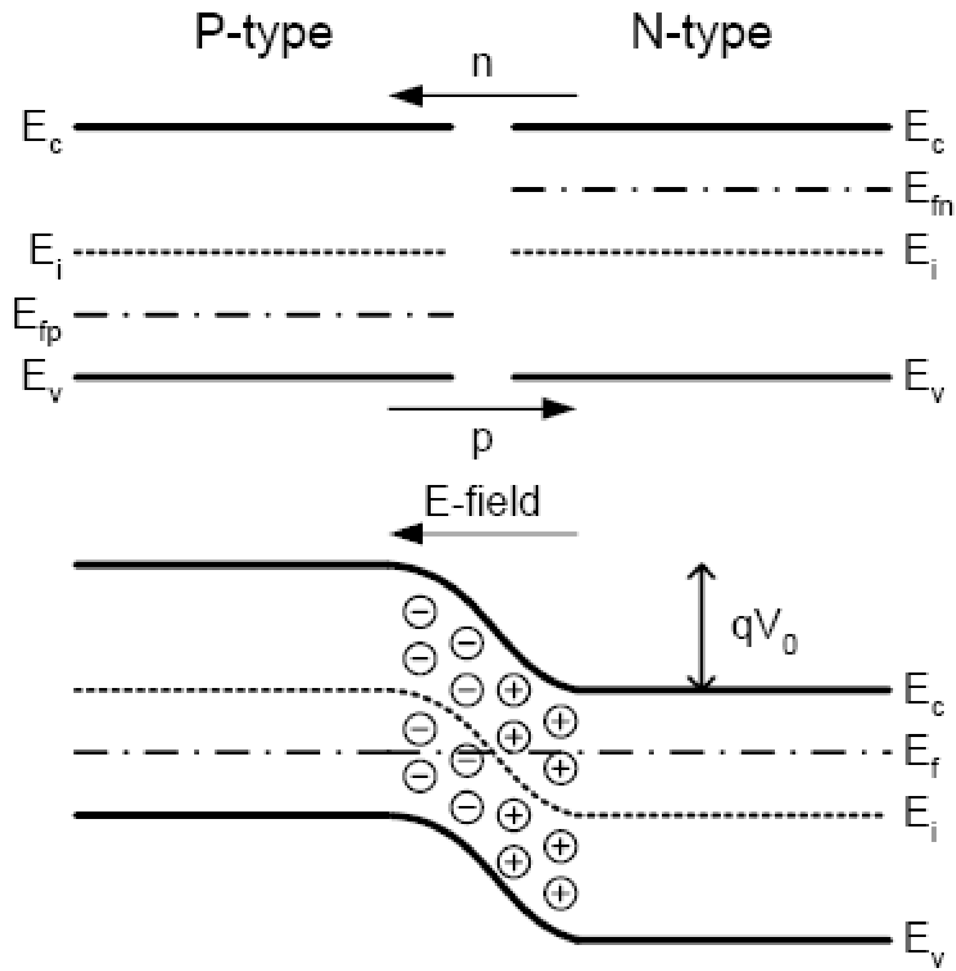
- Now let's look at physical mechanisms from which the current equations come.
  - We've seen that holes and electrons move through a semiconductor by two mechanisms – drift and diffusion



  - In equilibrium, diffusion current (I_D) is balanced by drift current (I_S). So, there is no net current flow. Drift current comes from (thermal) generation of hole-electron pairs (EHP).

Band diagrams

- When the P-type material is contacted with the N-type material, the Fermi levels ***must*** be at equilibrium.
- Band bending: The conduction and valence bands “bend” to align the Fermi levels.
- Electrons diffuse from the N-side to the P-side and recombine with holes at the boundary. Holes diffuse from the P-side to the N-side and recombine with electrons at the boundary. There is a region at the boundary of charged atoms – called the space-charge region (also called the depletion region b/c no mobile carriers in this region)
- An electric field is created which results in a voltage drop across the region – called the barrier voltage or built-in potential



What happens when n-type meets p-type?

- Holes diffuse from the p-type into the n-type, electrons diffuse from the n-type into the p-type, creating a diffusion current. The diffusion equation is given by

$$J_n = qD_n \frac{dn}{dx} \quad \text{where } D_n \text{ is the diffusion constant}$$

- Once the holes [electrons] cross into the n-type [p-type] region, they recombine with the electrons [holes].
- This recombination "strips" the n-type [p-type] of its electrons near the boundary, creating an electric field due to the positive and negative bound charges.
- The region "stripped" of carriers is called the space-charge region, or depletion region.
- V_0 is the contact potential that exists due to the electric field.

$$E(x) = -\frac{dV}{dx}$$

- Some carriers are generated (thermally) and make their way into the depletion region where they are whisked away by the electric field, creating a drift current.

Equilibrium motion of carriers

- In equilibrium, diffusion current is balanced by drift current. Moreover, the built-in potential (electric field) stops the diffusion by imposing a larger barrier to holes and electrons.
- The diffusion current is determined by the # of carriers able to overcome the potential barrier. The drift current is determined by the generation of minority carriers (in the depletion region) which then move due to the E-field. This generation is determined by the temperature.

$$J_n(x) = q\mu_n n(x)E(x) + qD_n\frac{dn(x)}{dx}$$

$$J_p(x) = \underbrace{q\mu_p p(x)E(x)}_{\text{drift}} + \underbrace{qD_p\frac{dp(x)}{dx}}_{\text{diffusion}}$$

- At equilibrium, the two components are equal…

$$\frac{D}{\mu} = \frac{kT}{q} \qquad \text{Einstein's relationship}$$

E-field and ‘built-in’ potential

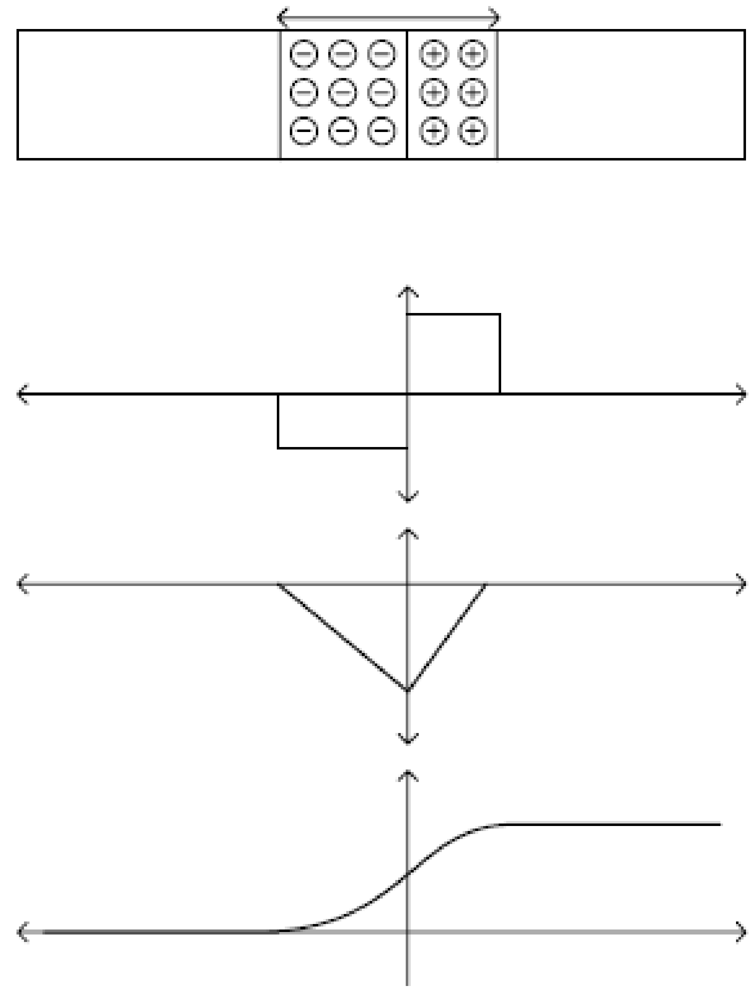
- Diffusion is balanced by drift due to bound charges at the junction that induce an E-field.
- Integrating the bound charge density gives us the E-field

$$\mathcal{E}(x) = \frac{1}{\epsilon_r \epsilon_0} \int_{-\infty}^{x} \rho(x)\, dx$$

- Integrating the E-field gives the potential gradient

$$\mathcal{E} = \frac{-dV}{dx}$$

$$V(x) = -\int_{-\infty}^{x} \mathcal{E}(x)\, dx$$



Junction built-in voltage

- With no external biasing, the voltage across the depletion region is:

$$V_0 = V_T \ln \frac{N_A N_D}{n_i^2}$$

  - Typically, at room temp, V_0 is 0.6~0.8V
    - How does V_0 change as temperature increases?

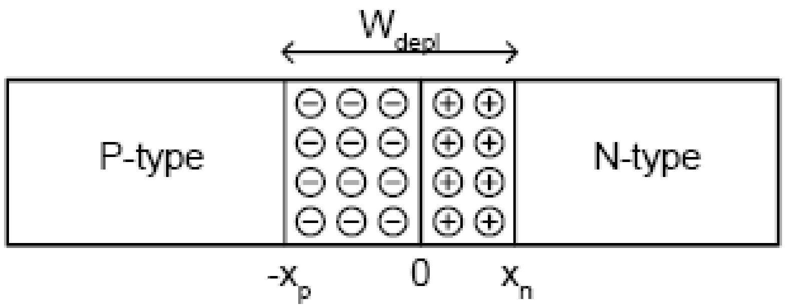
  - Interesting to note that when you try to measure the potential across the *pn* junction terminals, the voltage measured will be 0. In other words, V_0 across the depletion region does not appear across the diode terminals. This is b/c the metal-semiconductor junction at the terminals counteract and balance V_0 . Otherwise, we would be able to draw energy from an isolated *pn* junction, which violates conservation of energy.

Width of the depletion layer

- The depletion region exists on both sides of the junction. The widths in each side is a function of the respective doping levels. Charge-equality gives:

$$qx_pAN_A = qx_nAN_D$$

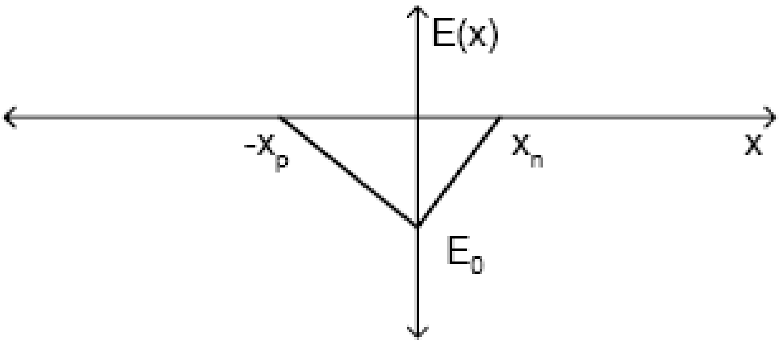
$$\frac{x_n}{x_p} = \frac{N_A}{N_D}$$



- The width of the depletion region can be found as a function of doping and the built-in voltage…

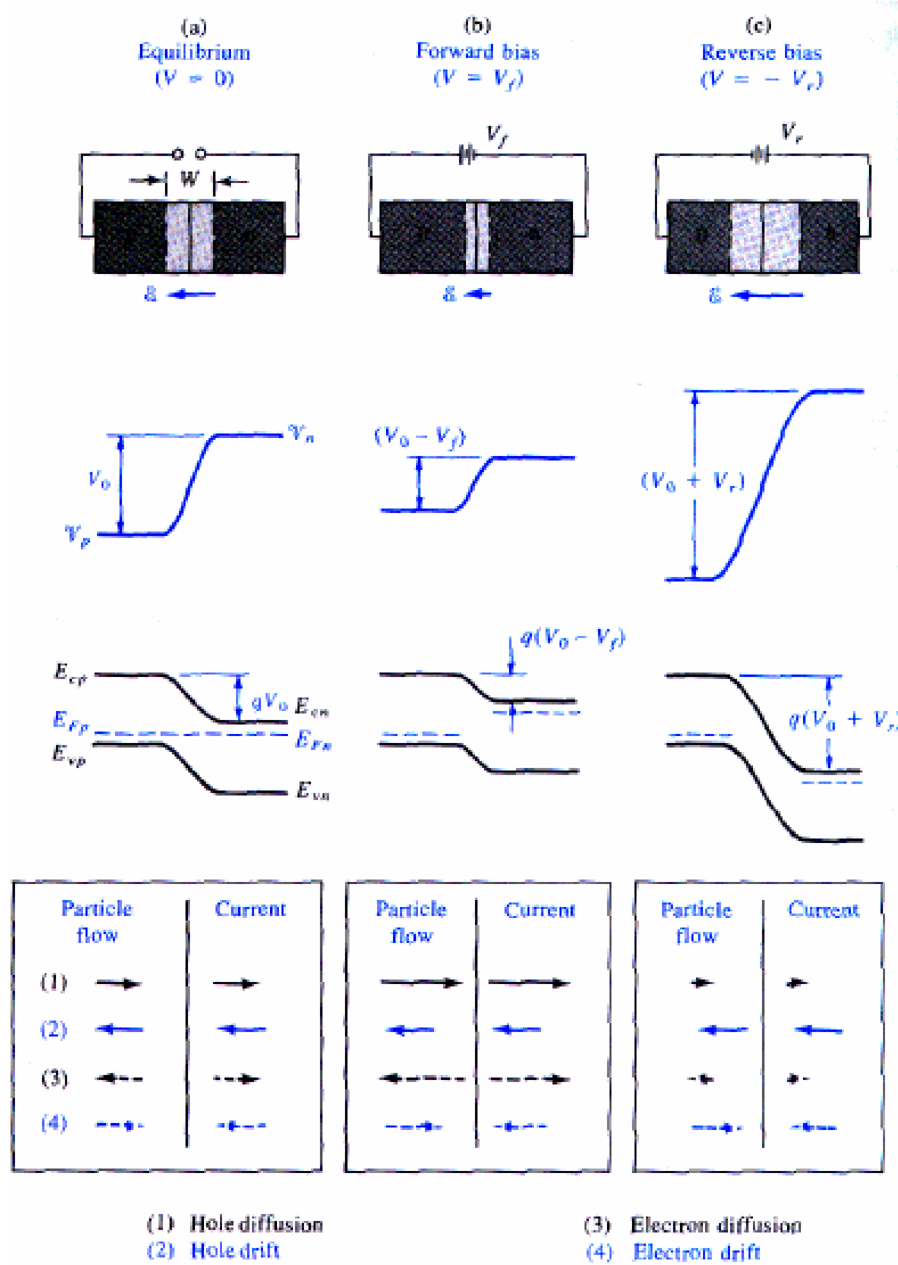
$$W_{depl} = x_n + x_p = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)V_0}$$

ε_s is the electrical permittivity of silicon = $11.7\varepsilon_0$ (units in F/cm)



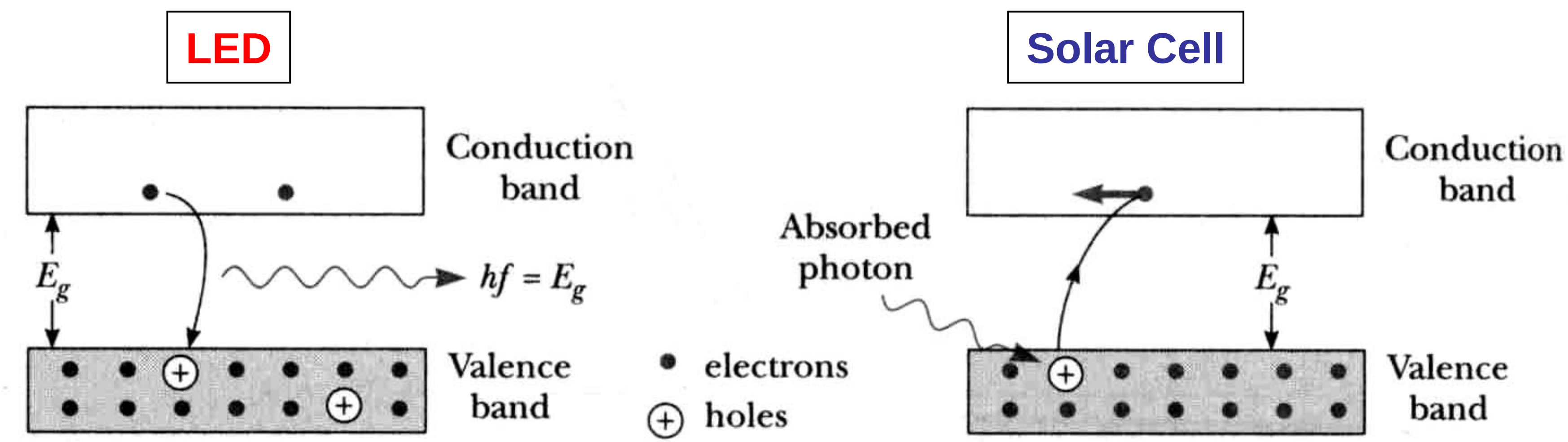
Band Diagram under Bias

- Applying a bias adds or subtracts to the built-in potential.
- This changes the diffusion current, making it harder or easier for the carriers to diffuse across.
- The drift current is essentially constant, as it is dependent on temperature.



PN Devices: LED and Solar Cell

- **Light-emitting diode (LED)**
  - **Converts electrical input to light output: electron in → photon out**
  - **Light source with long life, low power, compact design.**
  - **Applications: traffic and car lights, large displays.**
- **Solar Cell**
  - **Converts light input to electrical output: photon in → electron out (generated electrons are "swept away" by E field of pn junction)**
  - **Renewable energy source!**



(a) Light emission

(b) Light absorption