RECURSION

P.1. Design and Implement a recursive program in Pascal to work out the multiplication between two integers, "m" and "n" .

P.2. Design and Implement a recursive program in Pascal to work out the addition of the first "N" natural numbers.

P.3. Design and Implement a recursive program in Pascal to work out the exponentiation between two integers, "m" and "n," where "m" is the base and "n" the exponent.

P.4. Design and Implement a recursive program in Pascal to work out the maximun Common Divisor (MCD) of two integers greater than zero (Euclides algorithm´s) regarding the following properties of this function:

      2.1. if a>b, then MCD(a,b)=MCD(a-b,b)
      2.2. if a<b then MCD(a,b)=MCD(a,b-a)
      2.3. if a=b then MCD(a,b)=MCD(b,a)=a=b

P.5. Determine the output of this recursive program in Pascal:

```
Program invertir;
        Procedure invertirrec;
        Var
                c:char;
        Begin {invertirrec}
                Read(c);
                If c<>'.' Then
                        Begin
                                Invertirrec;
                                Write(c)
                        End {if}
        End; {invertirrec}
        Begin {invertir}
                Writeln('enter a chain of character ended by a dot');
                Invertirrec
        End. {invertir}
```

P.6. Design and implement a recursive program in Pascal to work out the binomial coeficients.

$$\left(\frac{n}{k} \right) = \frac{n!}{(n-k)!k!}$$

P.7. Design and Implement a recursive program in Pascal to work out the binary representation of an integer given its decimal representation. The header of the procedure has to be as follows:

PROCEDURE binary(num:integer);

P.8. Design and implement a recursive program in Pascal that reverse the digits in a natural number (inverting the order of the digits). The header of the procedure has to be as follows:
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PROCEDURE inversenumber(num:integer);

P.9. Design and implement a recursive program in Pascal that reverses the characters in a word (inverting the order of the characters). The header of the procedure has to be as follows:

PROCEDURE inverseword(w:string; n:integer);

P.10. Regarding the expression:

$$\int_a^b f(x)\,dx = \int_m^b f(x)\,dx + \int_a^m f(x)\,dx$$

(where m=(a+b)/2) Design and implement a recursive program in Pascal to work out the definite integral of the sin(x) function, dividing the interval [a,b] up to conforms the expression (|b-a|<ε) where ε represents the minimum size of the interval to consider the next expression true:

$$\int_a^b f(x)\,dx \approx (b-a)*f(m)$$